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ABSTRACT

This is the Teacher's Commentary for Mathematics for the Elementary School, Book 1 (Part 2), Special Edition. The writers have relied on the existing SMSG kindergarten and first grade materials as a framework. This special edition is designed to meet the needs of disadvantaged children. Included in the Commentary are background information for the teacher, discussion of activities in the text, and answers to activities and exercises. (RH)

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**SCHOOL**
**MATHEMATICS**
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# MATHEMATICS FOR THE ELEMENTARY SCHOOL

## BOOK 1 (Part 2)

*Teacher's Commentary*

SPECIAL EDITION (Revised)

U S DEPARTMENT OF HEALTH
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
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# MATHEMATICS FOR THE ELEMENTARY SCHOOL

## BOOK 1 (Part 2)

## *Teacher's Commentary*

SPECIAL EDITION (Revised)

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## Teachers' Commentary

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## Chapter 5

# RECOGNIZING GEOMETRIC FIGURES

## BACKGROUND

### INTRODUCTION

This chapter is devoted to geometry. The subject is introduced to the children by means of familiar three-dimensional shapes. This part of the discussion is very informal and the classification crude: objects are differentiated according to the shape.

The rest of the chapter, as well as the ensuing geometric material through the next several books, deals with plane geometry only. For convenience of reference we now outline the main ideas (even though many of them will not be encountered until later).

We shall study what may be called physical geometry--that is, the geometry of the world around us. The study involves a certain amount of abstraction, for the fundamental ideas we shall deal with are not things we can pick up or feel or see. We shall think of a point, for example, as an exact location in space. A point, then, has no size or shape or color; it has no physical attributes at all except its location. We indicate a point by making a pencil dot or a chalk dot, but agree that such a dot does not mark an exact location.

We may remark that the geometry studied in college courses is of a higher degree of abstraction still. There, the fundamental geometric objects like point and line are not defined at all, and the study proceeds deductively from certain formally stated assumptions about them (called axioms).

Our purpose here is to help the pupil observe and describe fundamental geometric relationships. The discussion is intuitive. In the primary grades we are not particularly concerned with formal deductions.

### POINT

By a point we mean an exact location--for example, the exact spot at the corner of a room where two walls and the ceiling meet. We indicate points by drawing dots; but we realize that a pencil dot, no matter how small, gives only an approximate location, not an exact one. (In fact, it is clear that a pencil dot on a sheet of paper covers infinitely many points--that is, more than can be counted.) Nevertheless, in order to keep the language simple, we refer to the dots themselves as actual points.

It is customary to denote points by capital letters. A point is a fixed location: points do not move. The point at the corner of the ceiling remains even if the whole building falls down. Nevertheless, it must be remembered that fixing a location is a meaningful notion only with respect to some particular frame of reference. Frames of reference in common usage are: the sun, the earth, a car, a person, a ruler. A point that is fixed with respect to one frame of reference need not be fixed with respect to a different one. For example, when a ruler is carried across the room, a point on the ruler remains fixed with respect to the ruler but does not remain fixed with respect to the earth.

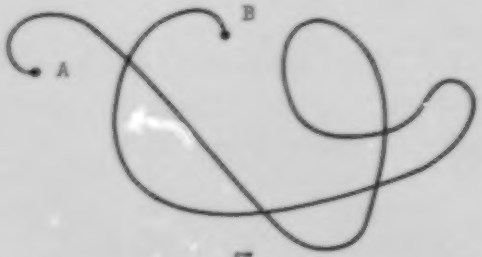
A geometric figure is any set of points.

## CONGRUENCE

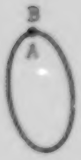
The idea of congruence in geometry is basic. Two geometric figures are said to be congruent provided one is an exact copy of the other. A test of congruency is whether one figure will fit exactly on the other. In practice, the objects may not be conveniently movable; then one tests for congruence by making a movable copy of one and checking it against the other. Of course, all such tests, since they involve actual physical objects, often including the human eye, are only approximate. Nevertheless, in order to keep the language simple, we shall say, "the segments $\overline{AB}$ and $\overline{CD}$ are congruent" (rather than seem to be)--just as people say, "Johnny and Jimmy are exactly as tall as each other" (rather than seem to be).

## CURVE

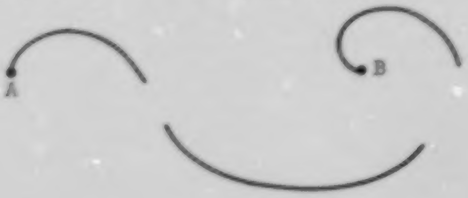
By a curve we mean any set of points followed in passing from a given point A to a given point B. Inherent in this definition is the intuitive notion of continuity; this is a curve.



and so is this:

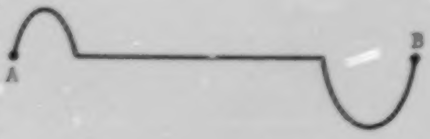


while this is not a curve because it has gaps, or is not continuous:

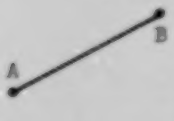


(However, it is a union of three curves.)

It is also noteworthy that, according to the definition, a curve can be straight (in contrast with everyday usage). This is a curve:



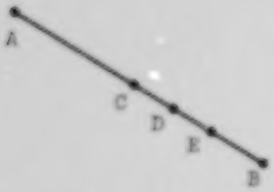
and so is this:



## LINE SEGMENT

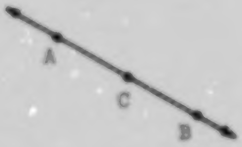
The last picture is an example of a line segment, that is, a straight curve. The endpoints are marked A and B; the line segment is denoted, accordingly, by either $\overline{AB}$ or $\overline{BA}$. Again, we agree that a single point is not a line segment.

Observe that a line segment can always be expressed in many different ways as a union of other line segments. For example, the line segment $\overline{AB}$ shown here is the union of the line segments $\overline{AC}$ and $\overline{CB}$, the union of the line segments $\overline{AD}$, $\overline{AE}$, and $\overline{CB}$, etc.



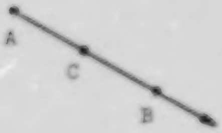
**LINE**

When a line segment is extended infinitely far in both directions, we get a line. Such extensions are only conceptual, of course, not practical. A line has no endpoints. No matter how far out we go in either direction along a line, still more of the line will lie ahead. The infinite extent is indicated by arrows. The line containing points A and B is denoted $\overleftrightarrow{AB}$. The line shown contains points A, B, and C; some names for this line are, therefore, $\overleftrightarrow{AB}$, $\overleftrightarrow{BA}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BC}$, etc.

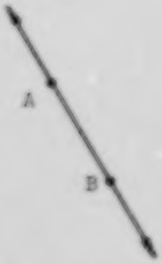


Note that, although $\overline{AB}$ and $\overline{AC}$ are different line segments, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are the same line.

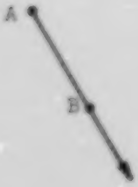
Just as a line is the infinite extension of a line segment in both directions a ray is the infinite extension of a line segment in one direction. A ray therefore, has a single endpoint. The infinite extent of a ray is indicated by an arrow. The ray with endpoint A and containing another point B is denoted by $\overrightarrow{AB}$. The endpoint must be written first. The ray shown has endpoint A and contains points B and C; some names for this ray are, therefore, $\overrightarrow{AB}$ and $\overrightarrow{AC}$.



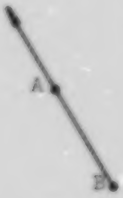
Note that, although $\overleftrightarrow{AB}$ and $\overleftrightarrow{BA}$ are the same line, $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.



$\overleftrightarrow{AB} = \overleftrightarrow{BA}$



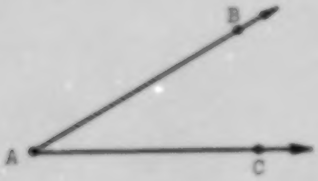
$\overrightarrow{AB}$

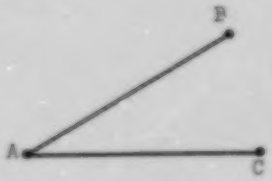


$\overrightarrow{BA}$

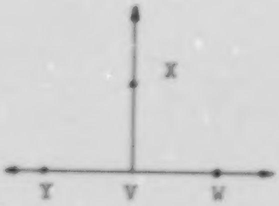
ANGLE

By an angle we mean the union of two rays having the same endpoint. (We exclude the case in which the two rays are part of the same line.) The common endpoint is called the vertex of the angle. The plural of "vertex" is "vertices". The angle formed by rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is denoted by $\angle BAC$ or $\angle CAB$. Two segments with a common endpoint determine an angle: segments $\overline{AB}$ and $\overline{AC}$ with common endpoint A determine the $\angle BAC$ with vertex A:



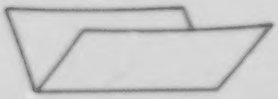


A special angle that makes frequent appearances in mathematics is a right angle. No formal definition of a right angle is given at this time. Instead, we will describe what is meant by a right angle in much the same way that you will convey the concept to your pupils.

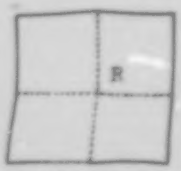


The drawing above represents two right angles, $\angle YVX$ and $\angle WVX$. This is one way of describing right angles. Two right angles are congruent and fit together to form a line. Also, in our example, if this page were folded along $\overrightarrow{VX}$, then $\overrightarrow{VY}$ and $\overrightarrow{VW}$ would coincide.

If a piece of paper were folded twice, as the drawing below indicates, and it were then unfolded, the creases suggest segments of two lines whose intersection is the point R. Thus R is the vertex of four right angles whose sides are extensions of appropriate pairs of creases.


Fold 1

Fold 2



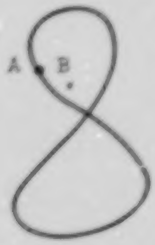
Unfolded with creases dotted

## PLANE

When a flat surface such as a table top, wall, or sheet of glass, or even this sheet of paper, is extended infinitely in all directions, we get a plane. Notice that if two points of a line lie in a given plane, then the entire line is contained in the plane. Two intersecting lines determine a plane. In the teaching material, the infinite extent of the plane is not stressed.

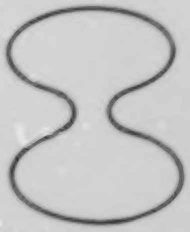
## CLOSED CURVE, SIMPLE CLOSED CURVE

We have called a curve any set of points followed in passing from a given point A to a given point B. When the points A and B coincide, the curve is said to be closed. For this level, we will consider only those curves which lie in the plane.



A closed curve

A closed curve that does not cross itself is simple

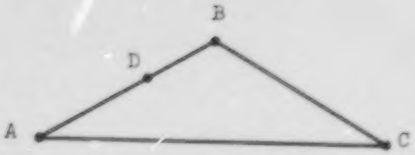


A simple closed curve

A simple closed curve has the interesting property of separating the rest of the plane into two subsets, an inside or interior (the subset of the plane enclosed by the curve) and an outside or exterior. Any curve connecting a point of the interior with a point of the exterior necessarily intersects the simple closed curve. (It may be of interest that this seemingly obvious fact is actually quite hard to prove.)

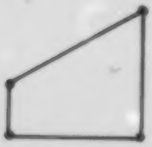
## POLYGON

An important class of simple closed curves is the class of polygons. A polygon is a simple closed curve that is a union of line segments. Recall that a line segment can always be expressed in many different ways as a union of line segments. Hence a polygon, too, can be expressed in different ways as a union of line segments.

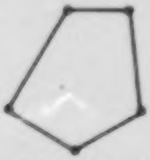


The union of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ =
the union of $\overline{AD}$, $\overline{DB}$, $\overline{BC}$, and $\overline{CA}$.

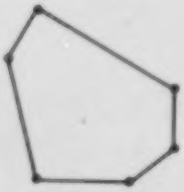
If we look at the various line segments in a polygon, we notice that they are of two kinds: those that are contained in other line segments, and those that are not contained in other line segments. For example, in the picture above, $\overline{AD}$ is of the first kind, since it is contained in the line segment $\overline{AB}$. On the other hand, $\overline{AB}$ is of the second kind, since it is not contained in any line segment except itself. Line segments of this second kind are called sides: a line segment in a polygon is called a side if it is not contained in any other line segment in the polygon. The polygon shown has three sides: $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$. A polygon of three sides is called a triangle. A polygon of four sides is a quadrilateral; of five sides, a pentagon; of six, a hexagon. (The last two names are not used in the teaching material.)



quadrilateral



pentagon



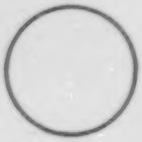
hexagon

The endpoints of the sides are the vertices (singular: vertex) of the polygon. The vertices of the triangle shown above are A, B, and C.

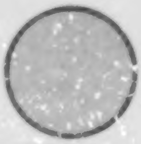
Rectangles are special kinds of quadrilaterals. Squares are special kinds of rectangles. A rectangle is a quadrilateral with four right angles. A square is a rectangle with four congruent sides.

## REGION

The union of a simple closed curve and its interior is called a region. We refer to a triangular region, rectangular region, or circular region, etc., indicating that the simple closed curve is a triangle, rectangle, or circle, etc. For example, an ordinary sheet of paper is a rectangular region; the edges of the paper form a rectangle.


circle

interior

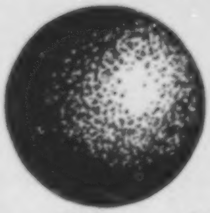

circular region

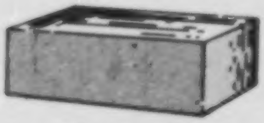
## 5-1. FAMILIAR THREE-DIMENSIONAL SHAPES

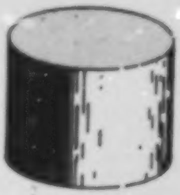
OBJECTIVE: To lead children to observe distinguishing features of spheres, rectangular prisms, and cylinders.

VOCABULARY: Shape, round, face, edge, corner, surface.

MATERIALS: A table on which there are familiar objects (at least 15): balls, boxes, blocks, plastic containers, and the like. These should be restricted to objects that can serve as models of spheres, rectangular prisms, and cylinders. A set of commercial models is highly recommended.


Sphere


Rectangular prism


Cylinder

### SUGGESTED PROCEDURE

### PRE-BOOK ACTIVITIES:

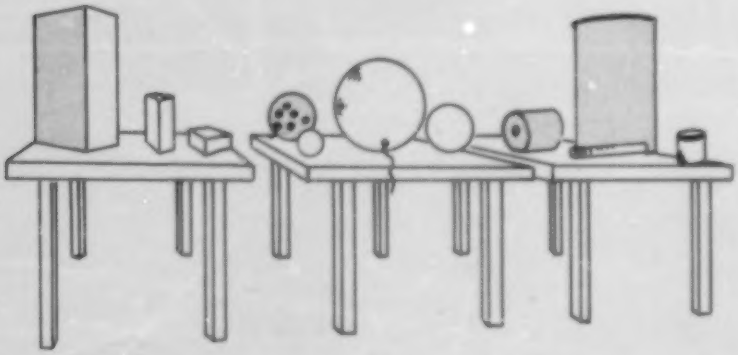
This exploratory lesson directs attention to the geometry of spheres, rectangular prisms, and cylinders. These terms, of course, will not be used with the children. There should be a sufficient number of objects (varied in color and shape) so that all children have an opportunity to handle and to discuss the objects. They should run their hands over surfaces along edges, etc. As the lesson proceeds, use the words object, item, and thing interchangeably so that the children will learn the word object.

You may begin this lesson by designating desks on which children are to place objects that have some kind of likeness to each other. Begin by asking a child to place an object (item or thing) on one of the desks. Ask another child to select a second object. If he does not think it should be placed with the first object, he may place it on another desk and explain in what way these objects are different. The classification has been established at this point.

When the other children place objects in the various sets, they should use this same classification. You may find that the first sorting is done according to color or size or use of the object or material from which it is made, etc. Let the children continue the classification by using six or more objects. As each object is placed with a set, discuss with the children whether or not it belongs with the other objects in the set.

Start again with all objects in one set and tell the children to think of other ways to sort them. Let the children develop several classifications. If shape has not been used as a basis for sorting, introduce it. First place a ball on one desk, a box on the next desk, and a can on the third. Then select another object and ask the children why it should be placed on a particular table. If a response is made that it has a shape like a ball, agree, and comment that it is a figure shaped like a ball.

The activity should result in some such arrangement as that pictured below.



After the sorting is completed, the children should identify what the objects in each set have in common. Their description of the sets may be: objects like boxes, objects like balls, objects like cans. Help develop the awareness of these shapes by describing the boxes as having edges, flat sides (faces), and corners; the cans as having edges (rims) but no corners; and the balls as having neither edges nor corners.

## USING THE PUPIL'S BOOK, PAGES 133-134: RECOGNIZING THE SHAPES OF FIGURES

### IDEAS

Objects are shaped in different ways. (Balls, cans, boxes.)

Page 133:

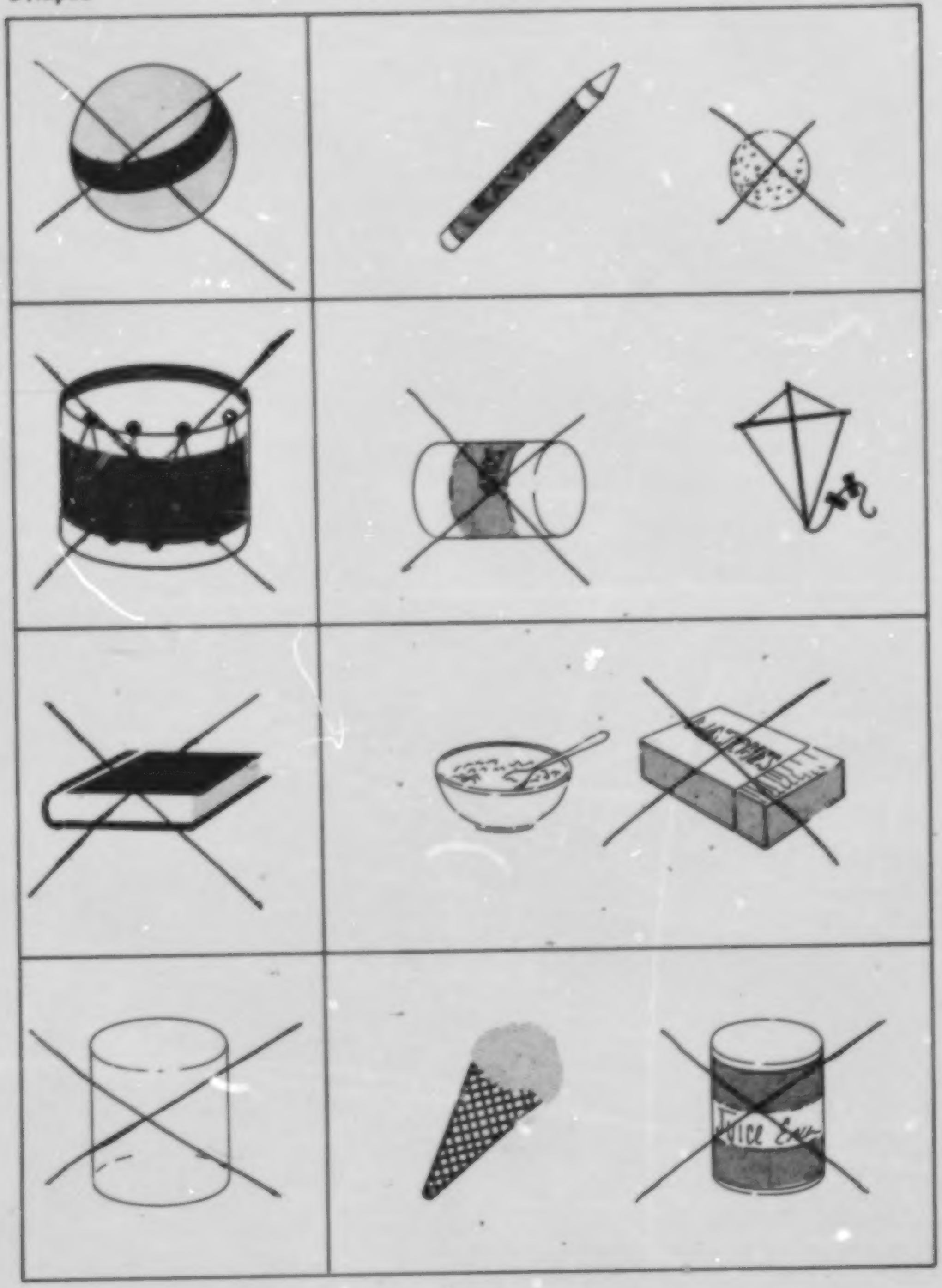
This is a teaching page. Call attention to some of the pictures of objects on the page. Ask the children to look at the first row. Note that a row goes across the page, not up and down. Ask what the first object in the first row is. (Ball.) Ask the children to make a mark, X, on the ball.

What are the names of the other pictures in the row? (Crayon, golf ball.)
Which picture has the same shape as the baseball? (Golf ball.)
Mark the golf ball in the same way the baseball is marked.

Then ask the children to mark, X, the first picture in the other rows, and one other picture shaped like the first one in the same row.

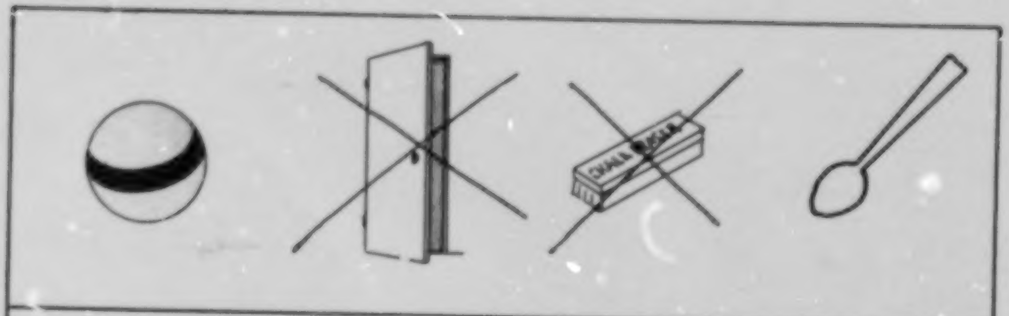
Page 134:

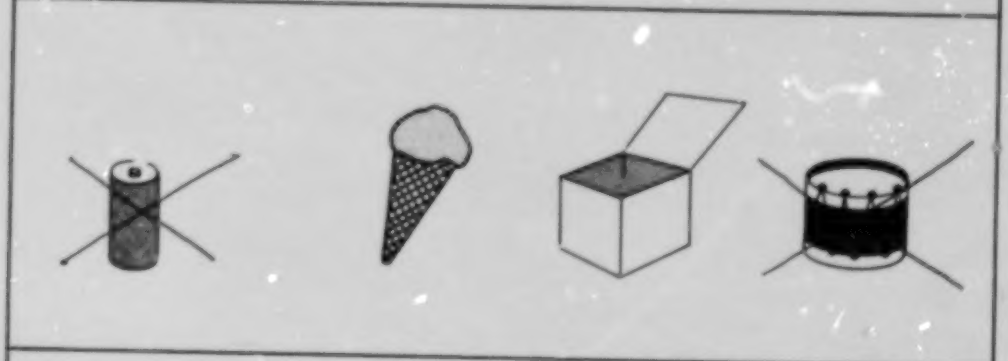
This is a teaching page. This page has more choices for marking in each row. Ask the children to look at the first row and mark the two objects that have the same shape. Check the accuracy of their markings, then give instructions for making the next row.

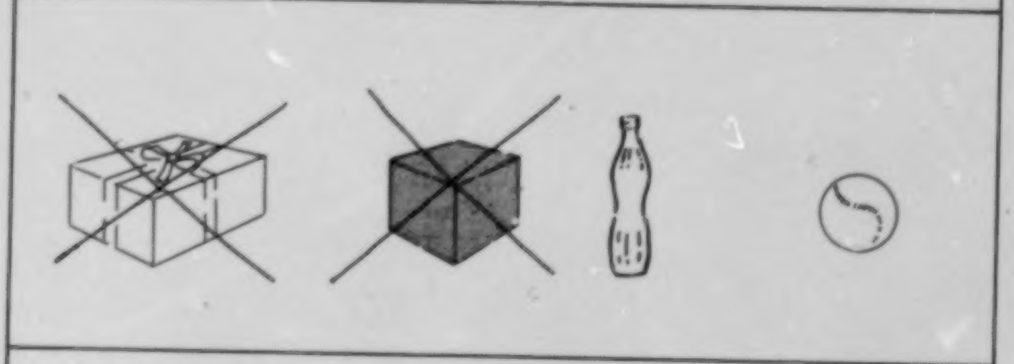


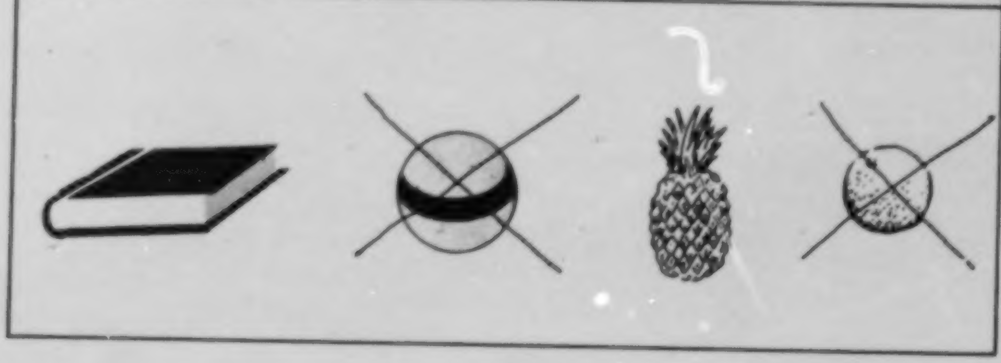
## Teaching Page

**Shapes**









FURTHER ACTIVITIES:

1. Ask a child to put his hands behind his back. Then place in his hands an object shaped like one of the three kinds in this lesson. (It would be advisable to include objects which had not been used in the earlier sorting.) Ask the child to identify its shape. Continue with other children and other objects. In each case, ask why the object is classified as it is. Chalk, dominoes, and cylindrical pinboxes would be helpful.

2. Have children identify other objects in the room that could be placed in one of the three categories. Children may wish to bring from home various objects to add to the collection. Flashlight batteries, balls, blocks, pencils, chalk, or simple toys can be classified as they are brought in.

3. Pictures of objects can also be brought in and classified. Have the children tell why each object can be put in that particular classification. This procedure not only helps to identify the geometric figures but also provides the association of the picture with the objects and with the geometric figure they represent. The pictures may be arranged on a bulletin board, in a scrapbook, etc.

4. If children ask the geometric names of the objects that they handle, supply these names whenever possible. Although introduction of such names as "rectangular prism", "cylinder", and "sphere" is not the purpose of this chapter, some children are interested in new words and will take pleasure in hearing them.

5. Select a child. Give him an object, such as a ball. Have him call another child to find another object in the room with the same shape. Continue with other objects and other children.

## 5-2. SIMPLE CLOSED CURVES

**OBJECTIVE:** A preliminary classification of some simple closed curves.

**VOCABULARY:** Straight, rounded, circle, inside, outside, on.

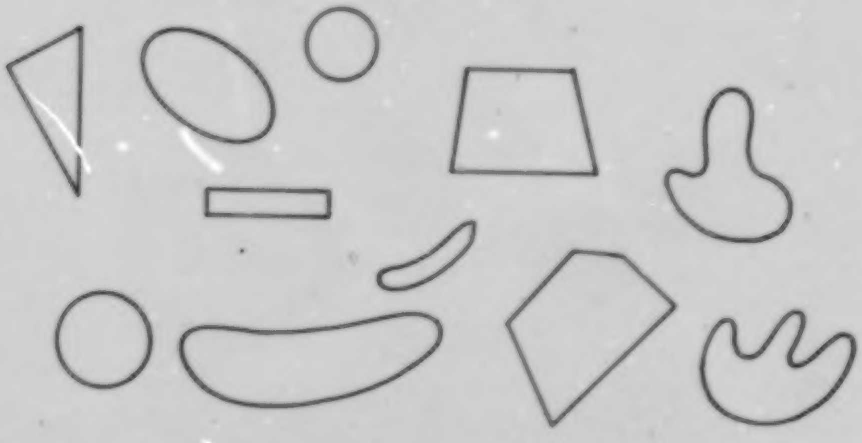
**MATERIALS:** Balls, boxes, and cans as in the preceding section; models of circles, triangles, rectangles, and other curved or polygonal figures, such as triangles from rhythm instruments, rectangular picture frames, circular embroidery hoops, rubber bands, stretched around pegs on a pegboard (or nails in a piece of ceiling tile), models made from wire or starched string (do not use cardboard sheets as they suggest the regions rather than the curves themselves); chalk and string for drawing circles on the chalkboard.

### SUGGESTED PROCEDURE

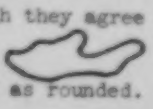
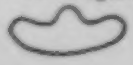
### PRE-BOOK ACTIVITIES:

### DISTINGUISHING BETWEEN "STRAIGHT" AND "ROUNDED"

Before the lesson, draw several polygons and other simple closed curves on the chalkboard. Include at least two circles.



Point out that some of the figures are rounded, while others have straight sides. Discuss and classify each figure in turn.

Children instinctively identify a representation of a circle as rounded. Indeed, they tend to believe that any geometric figure which does not represent a circle is not rounded even though they agree that the figure does not have straight sides. Figures such as  or  are particularly difficult for children to identify as rounded.

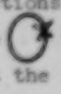
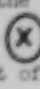
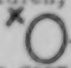
If this difficulty does arise, it is suggested that a book be classified as having straight sides. This then may be used as a model of straightness. Place the book edge along the side of the figure. If the side of the figure does not coincide with the model, then the figure is classified as rounded.

Display and discuss the triangles, frames, and hoops, and the pegboard and wire models. Use these representations of geometric figures to differentiate the outline from its interior and exterior. Ask the children to place a finger inside, outside and on the outline or figure.

Display the balls, boxes, and cans. Show the circular seam of a ball. (Do not use a baseball; its seam is not a plane curve.) Indicate the rounded rims of the cans. Point to the straight edges of the boxes.

Have the children look for objects about the room whose shapes they can classify: the rounded rim of the wastebasket or clock, the straight edges of the desk or window, etc.

USING THE PUPIL'S BOOK, PAGE 135: ROUNDED OR STRAIGHT

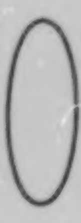
This is a teaching page. Read the instructions to the children, the child is to make a mark somewhere on the figure  not  or . Have one child do the marking on a book displayed at the front of the room while the other children work individually.

Teaching Page

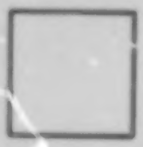
## Rounded or Straight

Mark each rounded figure blue.

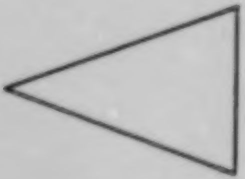
Mark each figure with straight sides red.



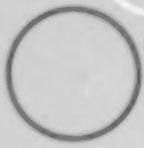
blue



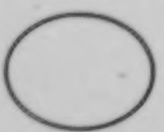
red



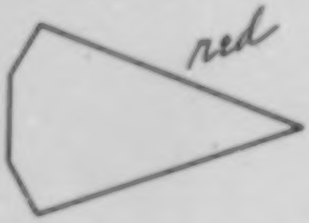
red



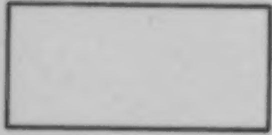
blue



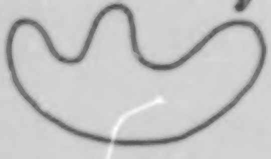
blue



red



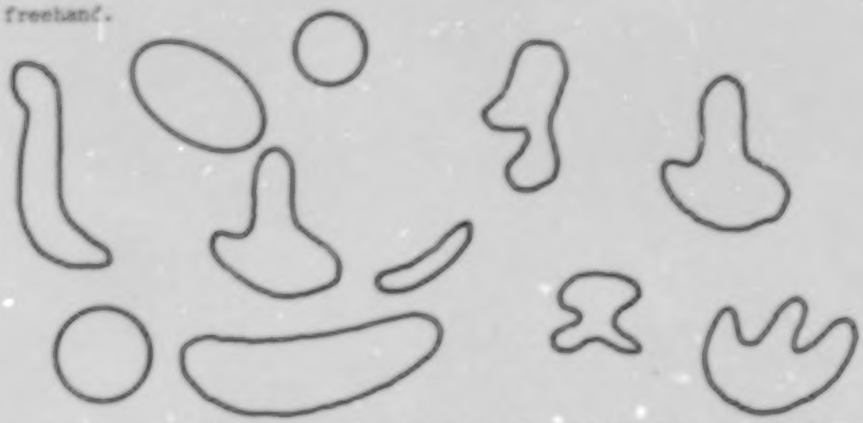
red



blue

## DISTINGUISHING CIRCLES FROM OTHER ROUNDED SHAPES

Direct children's attention again to the figures on the chalkboard. Tell the children that you are going to erase all the figures (or pictures) with straight sides. Have them pick out the figures for you. When all the polygons have been erased, replace them with curved figures that you can draw freehand.



Introduce the word circle. Consider the figures one by one, picking out the circles. Have the children tell why the circle is special. ("It looks the same from every direction", etc.)

Before erasing the representations of circles, make it clear that the circle is the outline. Ask various children to come to the chalkboard and place a finger inside, outside, and on the figure.

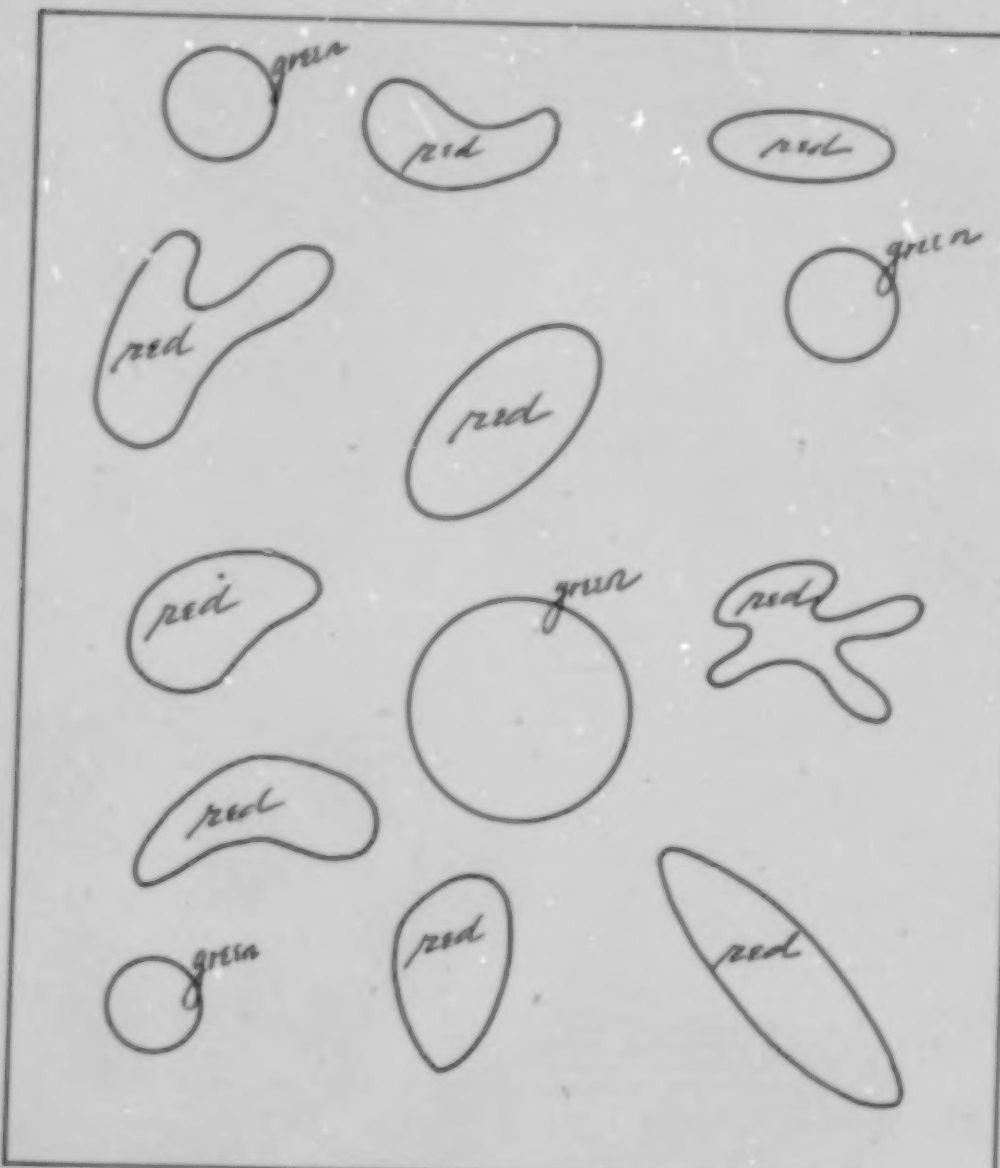
## USING THE PUPIL'S BOOK, PAGE 136: CIRCLES

Read the instructions to the children. The pupil is to make a mark somewhere on the figure.

## Circles

Mark each circle green.

Color the inside of the other figures red.


green
red
red
red
green
red
red
green
red
red
green
red
red

## 5-3. POLYGONS

**OBJECTIVE:** A preliminary classification of some polygons.

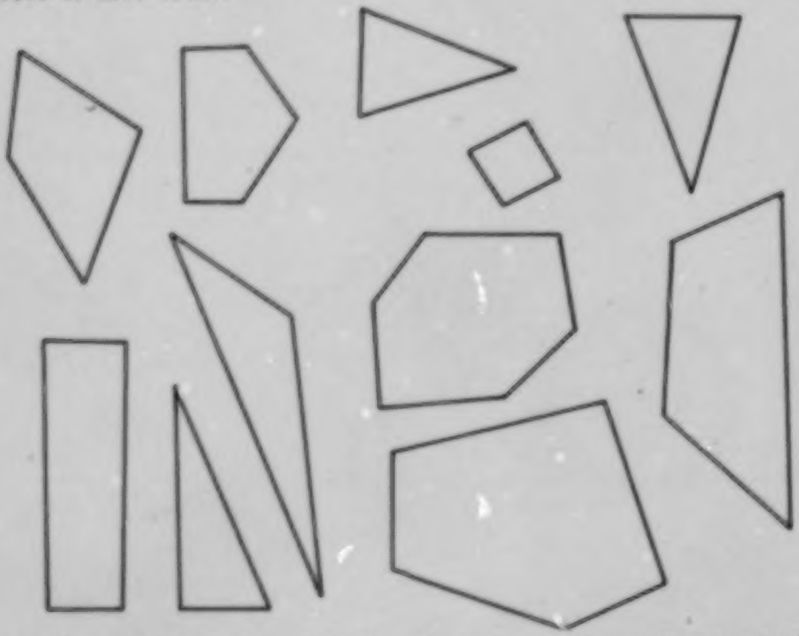
**VOCABULARY:** Triangle, rectangle, square.

**MATERIALS:** Boxes, models of triangles, rectangles, and other polygons, such as triangles from rhythm instruments, rectangular picture frames, rubber bands stretched around pegs on a pegboard (or nails in a piece of ceiling tile), models made from wire or starched string; sticks of various lengths.

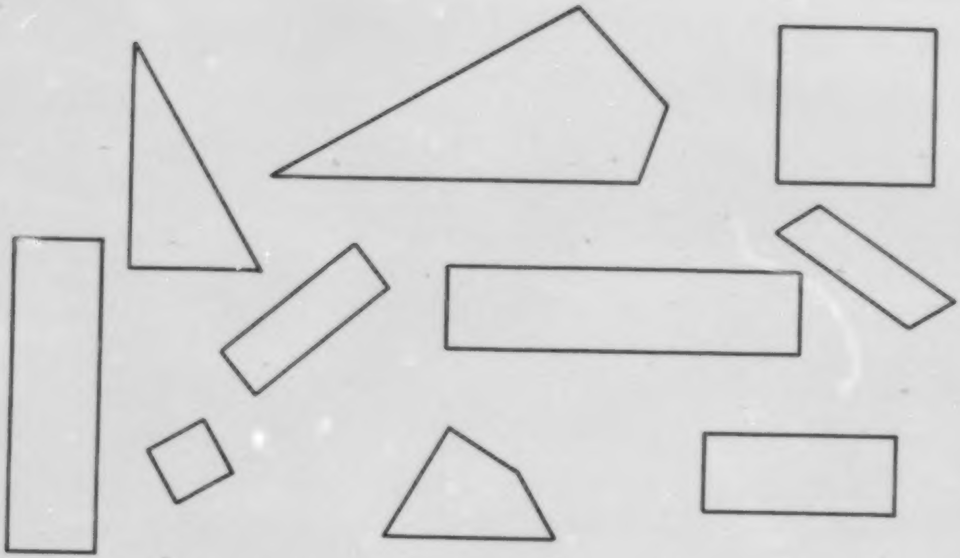
### SUGGESTED PROCEDURE

### PRE-BOOK ACTIVITIES:

This lesson requires some preparation of the chalkboard. On the left side of the chalkboard, draw several polygons. (As shown below). Include at least three triangles and three quadrilaterals, and a few polygons with five or more sides.



On the right side of the chalkboard, draw several quadrilaterals (as shown below). Include at least five rectangles, two of which are square; at least two of the rectangles, including one of the squares, should be "tilted". Keep this section covered from view until needed.



## CLASSIFYING POLYGONS ACCORDING TO THE NUMBER OF SIDES

Ask the class how the set of figures (or pictures) drawn here differs from those discussed last time. (All of these have straight sides.) Pick out a triangle and show that it has three sides; write "3" inside the triangle. Pick out a quadrilateral and show that it has four sides; and write "4" inside. Then consider the remaining figures in turn, getting the children to agree on the number of sides, and recording the number inside the figure.

See if children know the name, triangle, for polygons having exactly three sides. Suggest the name if necessary. Consider the figures once more, picking out the triangles. The word "quadrilateral" is not introduced at this stage, but should be given if a child asks for the name of a polygon of four sides. For five or more sides, it is enough to tell the children that special names do exist.

Display the metal triangles, the picture frames, and the pegboard and wire models of polygons. Have the children classify their shapes.

Supply sticks of various lengths for the children to form into triangles. Make sure that the two shortest sticks have a combined length greater than the longest; then the child will always be able to construct a triangle.

If you wish to discuss the number of angles of a triangle or any other polygon, it is better to use the term "points where the sides meet" since it is difficult to define an angle correctly at this grade level.

USING THE PUPIL'S BOOK, PAGE 137: NUMBER OF SIDES

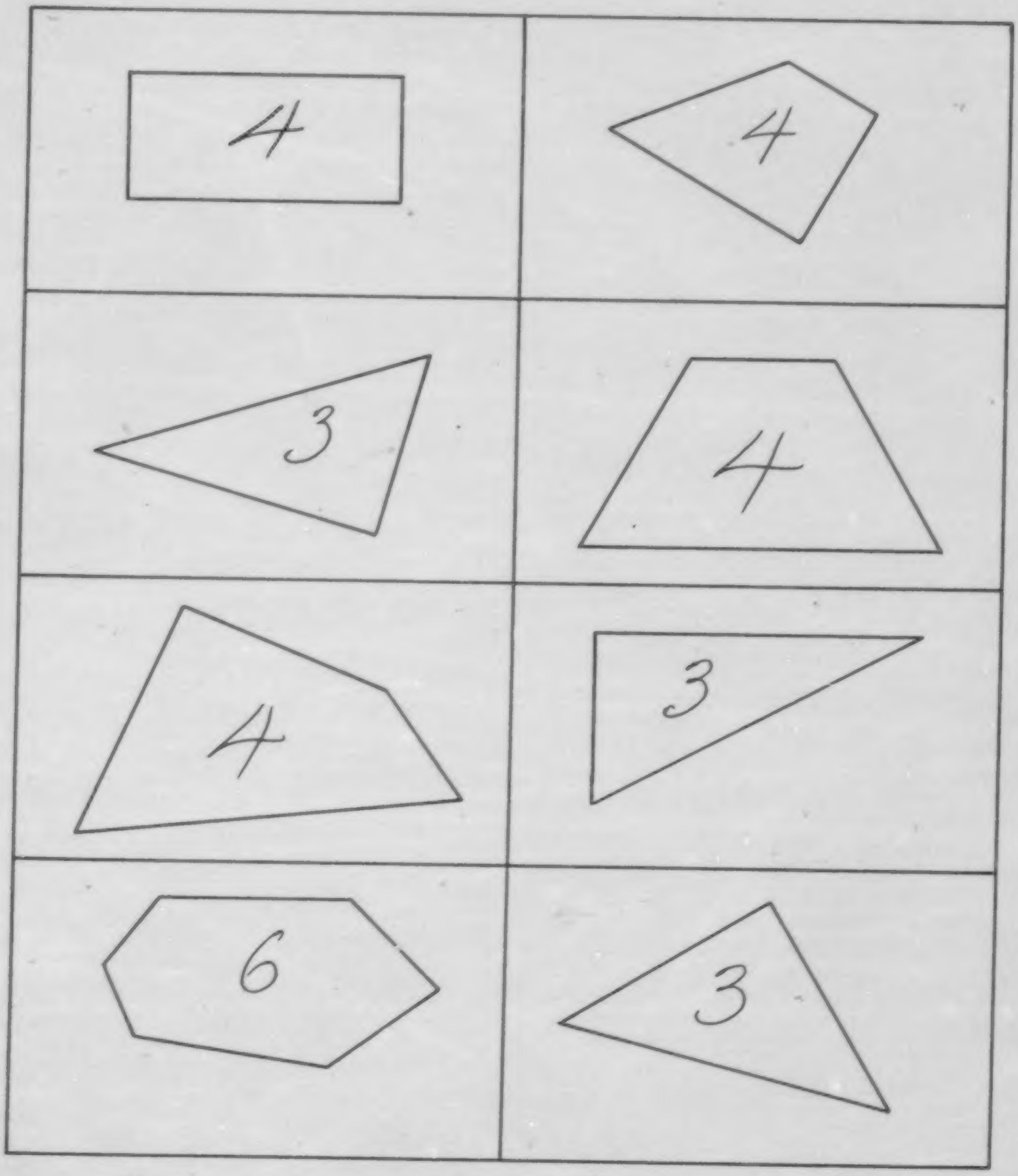
Read the instructions to the children.

Page 138: TRIANGLES

Ask the children to make a blue mark on any figure that represents a triangle.

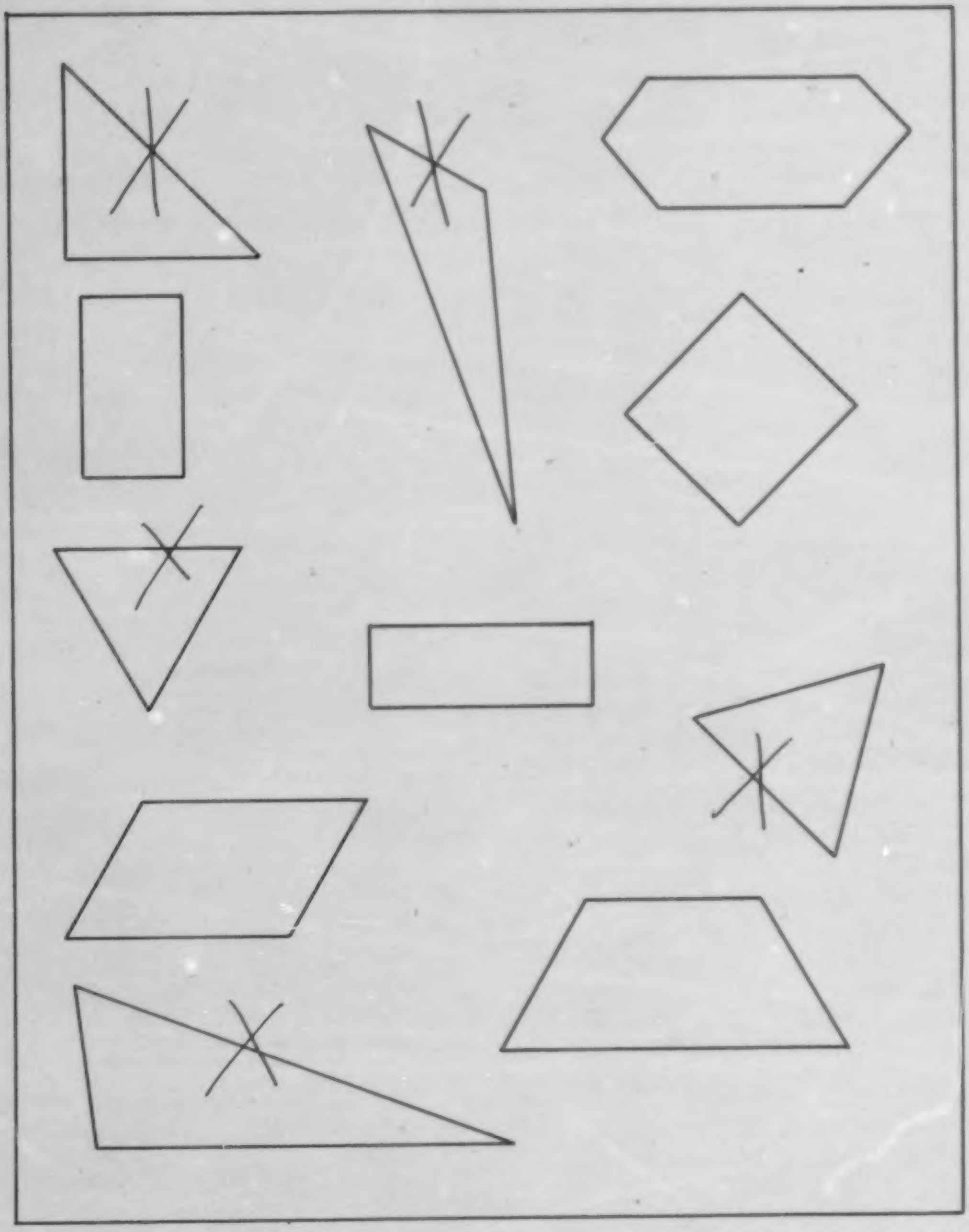
## Number of Sides

Write the number of sides inside each figure.



## Triangles

Mark each triangle blue.



## DISTINGUISHING RECTANGLES FROM OTHER QUADRILATERALS

Disclose the figures on the right side of the chalkboard. Ask the class how the set of figures (or pictures) drawn here differs from the set discussed earlier in this section. (Each of these has exactly four sides.) Tell the class you are all going to look for some special figures in the set. Ask whether some child sees a figure that is special in any way. Point to the rectangular picture frame and the rectangular window frame as examples of the special shapes we are looking for. If necessary, ask explicitly about the corners. Try to lead the children to the idea that in a rectangle, all four corners "look alike". If any difficulty arises in determining if the four corners "look alike", use a paper model . If the paper model fits the corners of the figure exactly, this will be accepted as meaning that the four corners "look alike." Introduce the word "rectangle."

Some children may object to calling the square a rectangle; point out that it is a special kind of rectangle, just as a lollipop is a special kind of candy.

Have the children make rectangles by bordering a sheet of paper with a crayon.

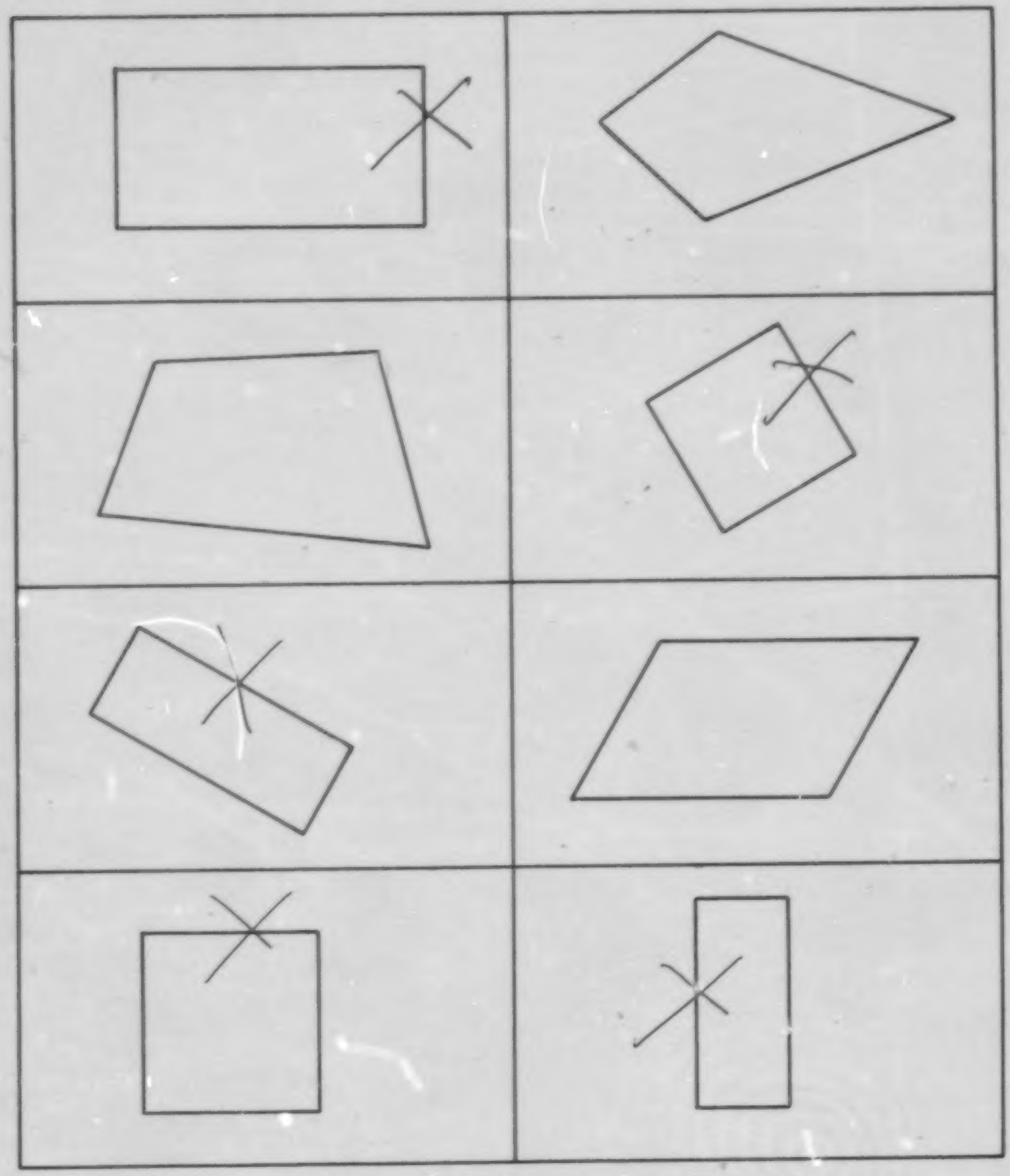
Display several boxes and point out how their edges form rectangles. Have the children look for rectangles in the room as boundaries of desks, the chalkboard, and so on.

## USING THE PUPIL'S BOOK, PAGE 139: RECTANGLES

Read the instructions to the children. The child is to make a mark somewhere on the figure.

## Rectangles

Mark each rectangle red.



## DISTINGUISHING SQUARES FROM OTHER RECTANGLES

Draw several squares and other rectangles on the chalkboard before the lesson begins. Make certain that the rectangles, other than squares, can be clearly distinguished from squares. Ask the class if all of the figures have four sides. Then ask if all the figures have corners that "look alike." If there is any question about this point, use the folded paper model to show that the model will fit the corners of the figures exactly. Ask for the name of these figures. (Rectangles) Tell the class you are going to look for the special kind of rectangles. The children will name this figure as a square. Use a piece of string to demonstrate that the sides of the figure are congruent. Do not use this term but merely place the string along one of the sides and show that this piece of string will fit exactly on all of the other three sides. Use this same procedure with another figure that is a rectangle but not a square to show that this is not true of rectangles that are not squares.

Display several boxes and point out how their edges form squares or other rectangles. Have the children look for squares and other rectangular figures in the room.

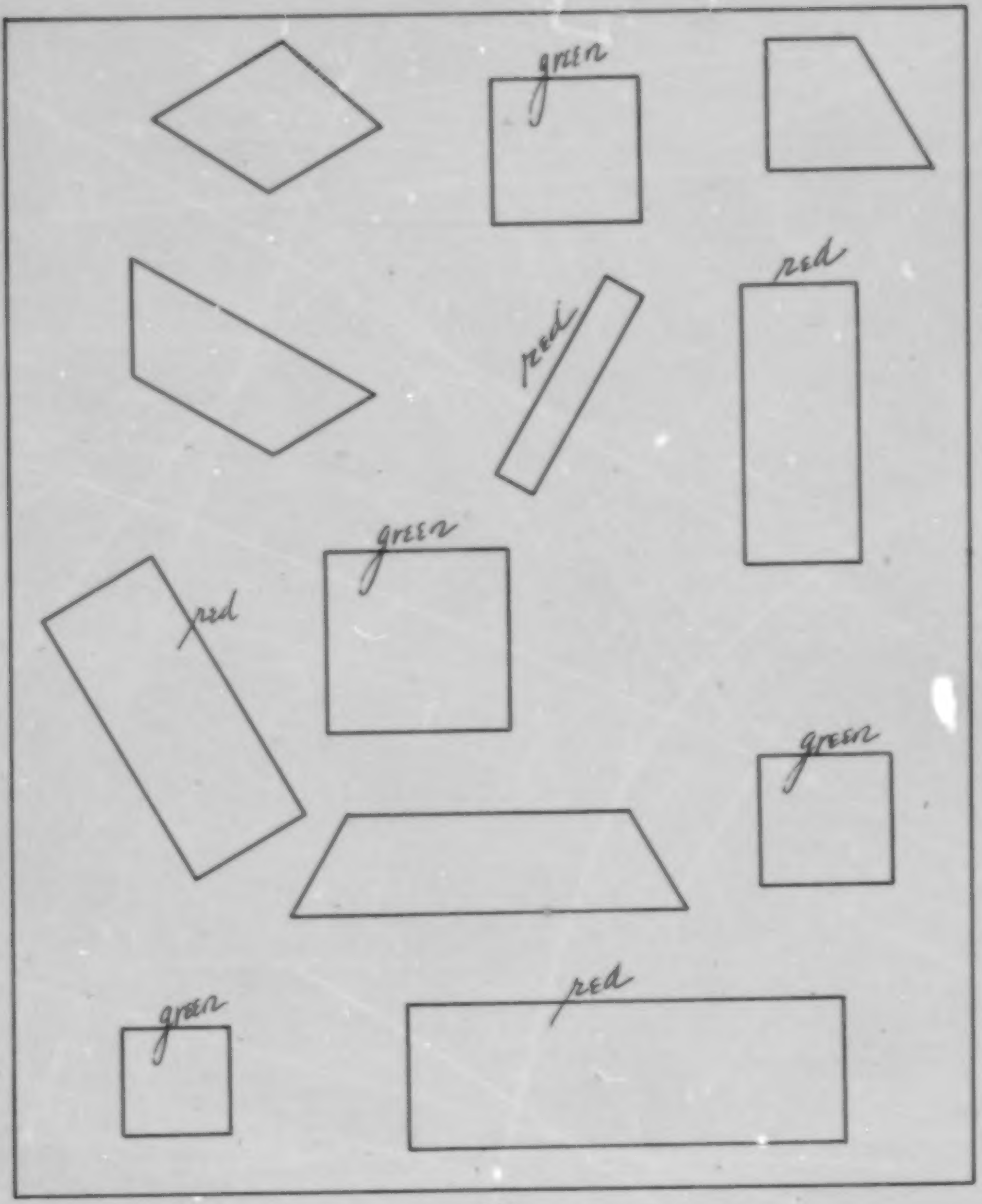
USING THE PUPIL'S BOOK, PAGE 140:

Read the instructions to the children.

## Squares and Other Rectangles

Mark each square green.

Mark all other rectangles red.


green
red
red
green
red
green
green
red

## 5-4. CLASSIFYING REGIONS

**OBJECTIVE:** To recognize that a circular region, rectangular region, etc., consists of the curve itself plus its interior.

To identify circular, rectangular, triangular, and square regions.

**VOCABULARY:** Circular region, rectangular region, triangular region, and square region.

**MATERIALS:** Wire models of circles, rectangles, squares, triangles; flannel regions of the same shapes.

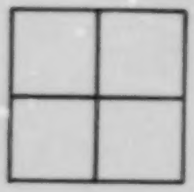
### SUGGESTED PROCEDURE

### PRE-BOOK ACTIVITIES:

In preparation for study of regions, review the ideas of inside, outside, and on. In the SMSG Kindergarten book there are many activities that call attention to these ideas.

The use of playground circle games can reinforce the idea of a circle through their references to the above terms: Such games include: "Froggie in the Middle", "The Farmer in the Dell", "Bow Belinda", "In and Out the Window", "Looby Loo", "The Old Brass Wagon", and "Hokey Pokey". Step on the circle to show where the curve is.

The playground outlines for "Four Square" can be used to find several squares.



The outlines of the volleyball or basketball court are examples of rectangles, though these may tend to be too large for delineation at this time.

On the flannel board place an assortment of regions of the types above. Compare these with models of circles, rectangles, triangles, and squares. Ask how a circular figure is like a circle and how it is different. (Alike in shape; the edge of the felt figure is like the wire circle; the inside of the felt figure is "full"; and so on.)

Tell the children that any object like the felt cutout has a longer name. It is called a circular region. Its edge is a circle.

Continue with the other figures. Refer to their straight edges as sides. Use the terms triangular region, rectangular region, and square region.

Place the wire models on a table in separate classifications. Ask a child to go to the flannel board, remove a region, compare it with a wire model, name the region, and place it in the proper classification. Continue until all the figures have been removed and classified.

Some children will use the terms: circle region, rectangle region, and triangle region. Agree and state, "Yes, that is a circular region." If the teacher consistently presents the correct language pattern, the children will, in time, use the proper terminology.

## USING THE PUPIL'S BOOK, PAGES 141-146: REGIONS

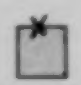
### Ideas

A circular region, rectangular region, etc., consists of the curve itself and its interior.

### Pages 141-144:

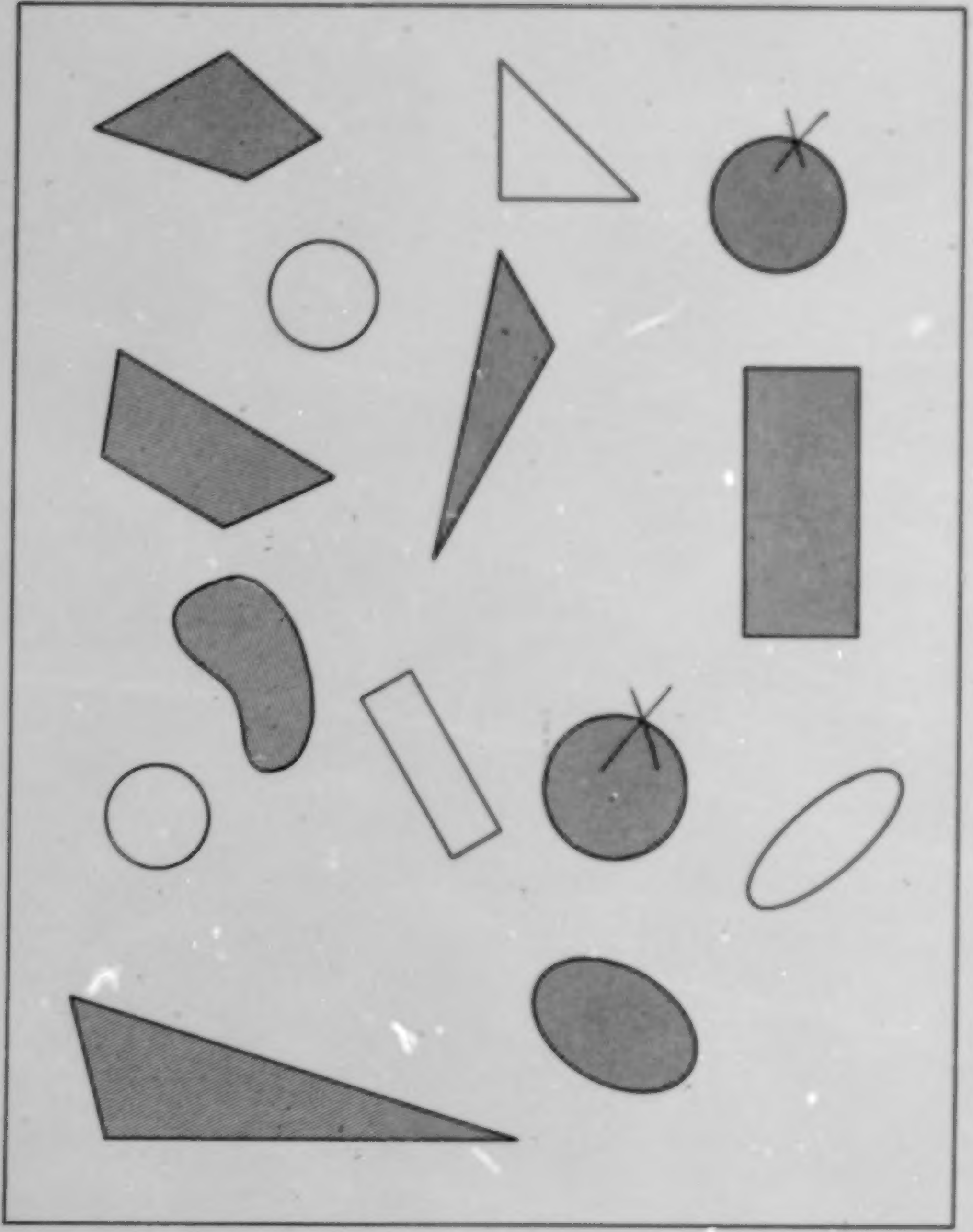
Each page includes a different type of region to classify. The instructions should be read to the children.

### Pages 145-146:

Here the children need to mark the curve itself: 
Crayons should be used. Red if the curve is a circle; blue if a triangle; green if a square; and black if it is a rectangle that is not a square.

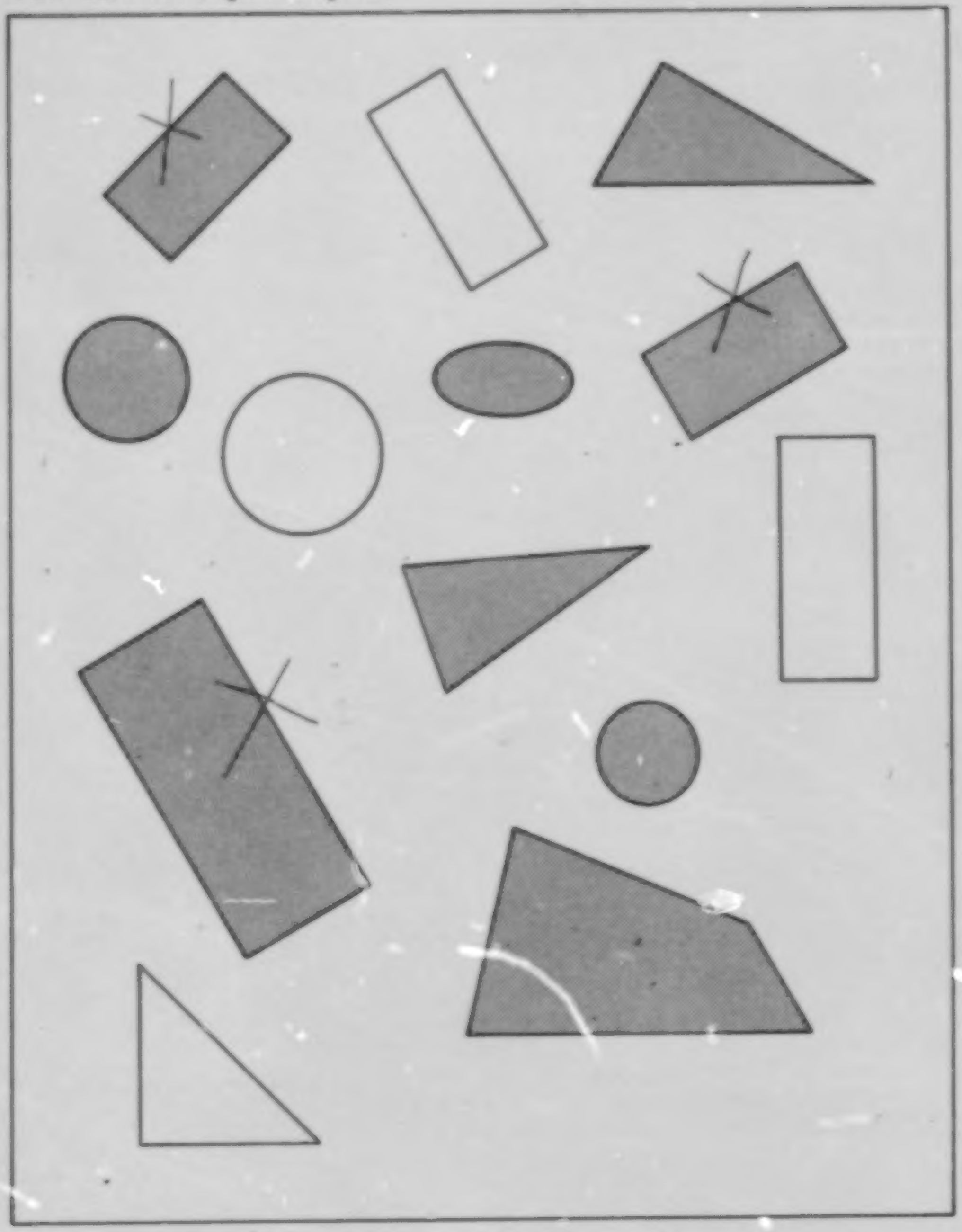
## Regions

Mark the circular regions.



## Regions

Mark each rectangular region.



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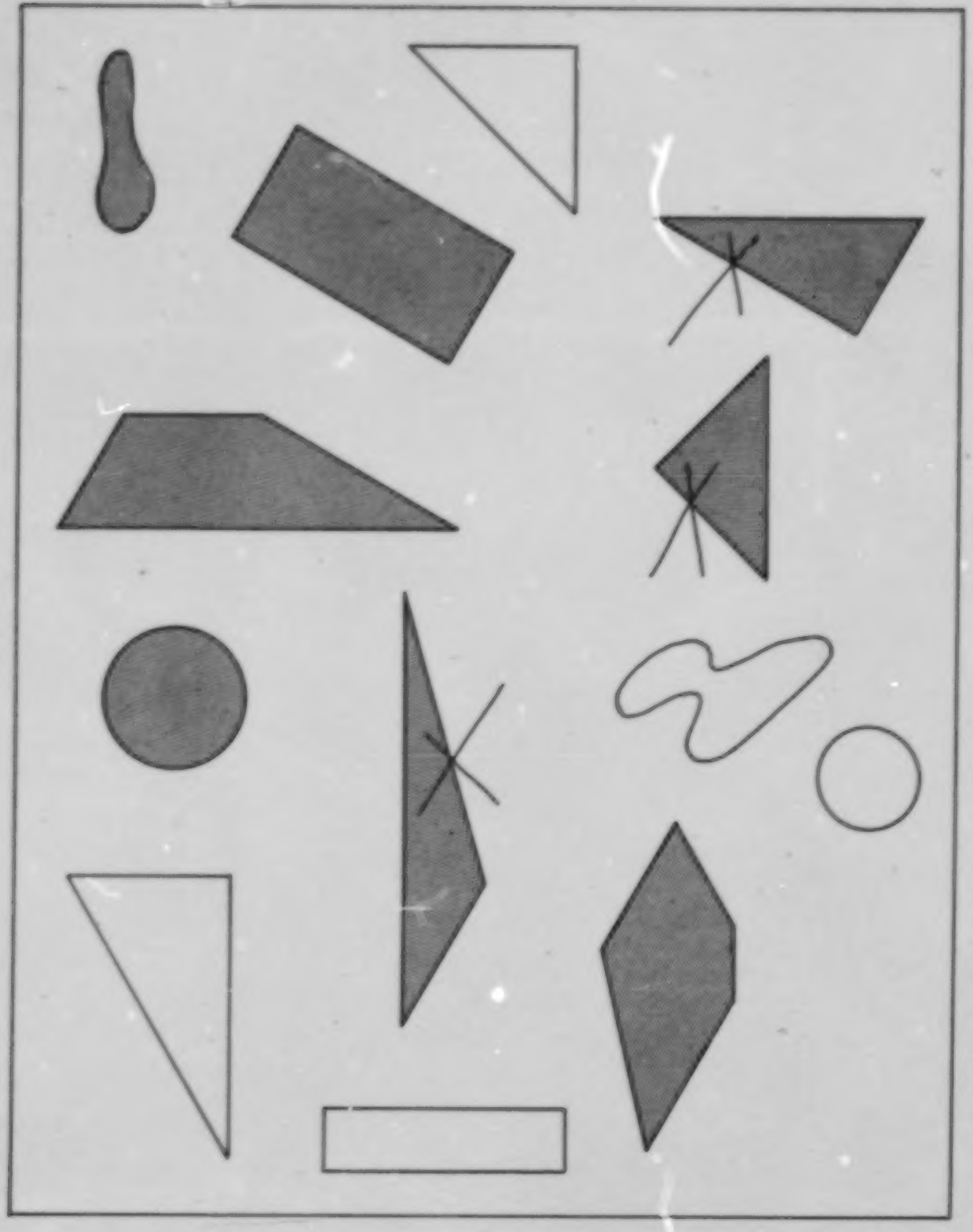
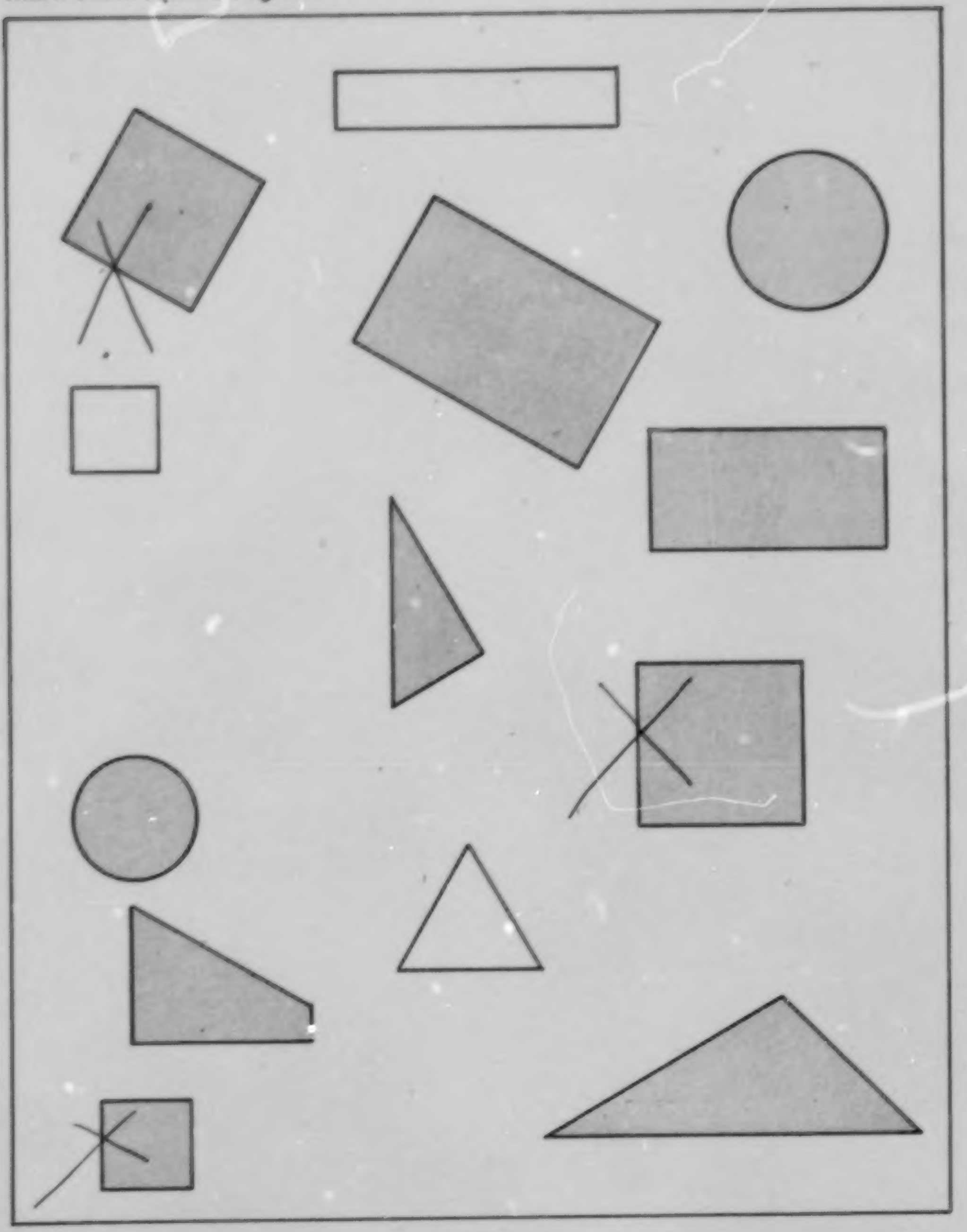
## Regions

Mark the triangular regions.



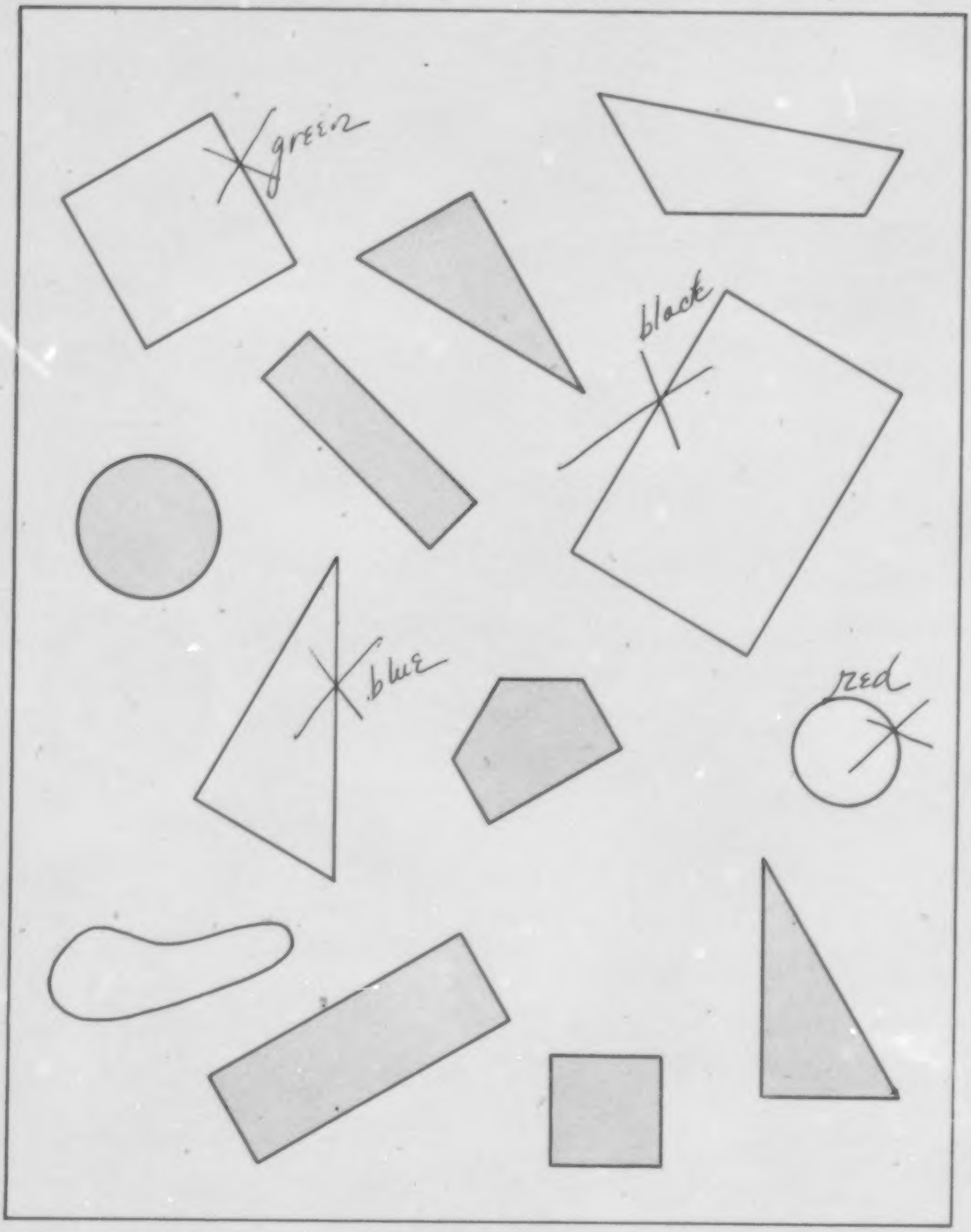
## Regions

Mark each square region.



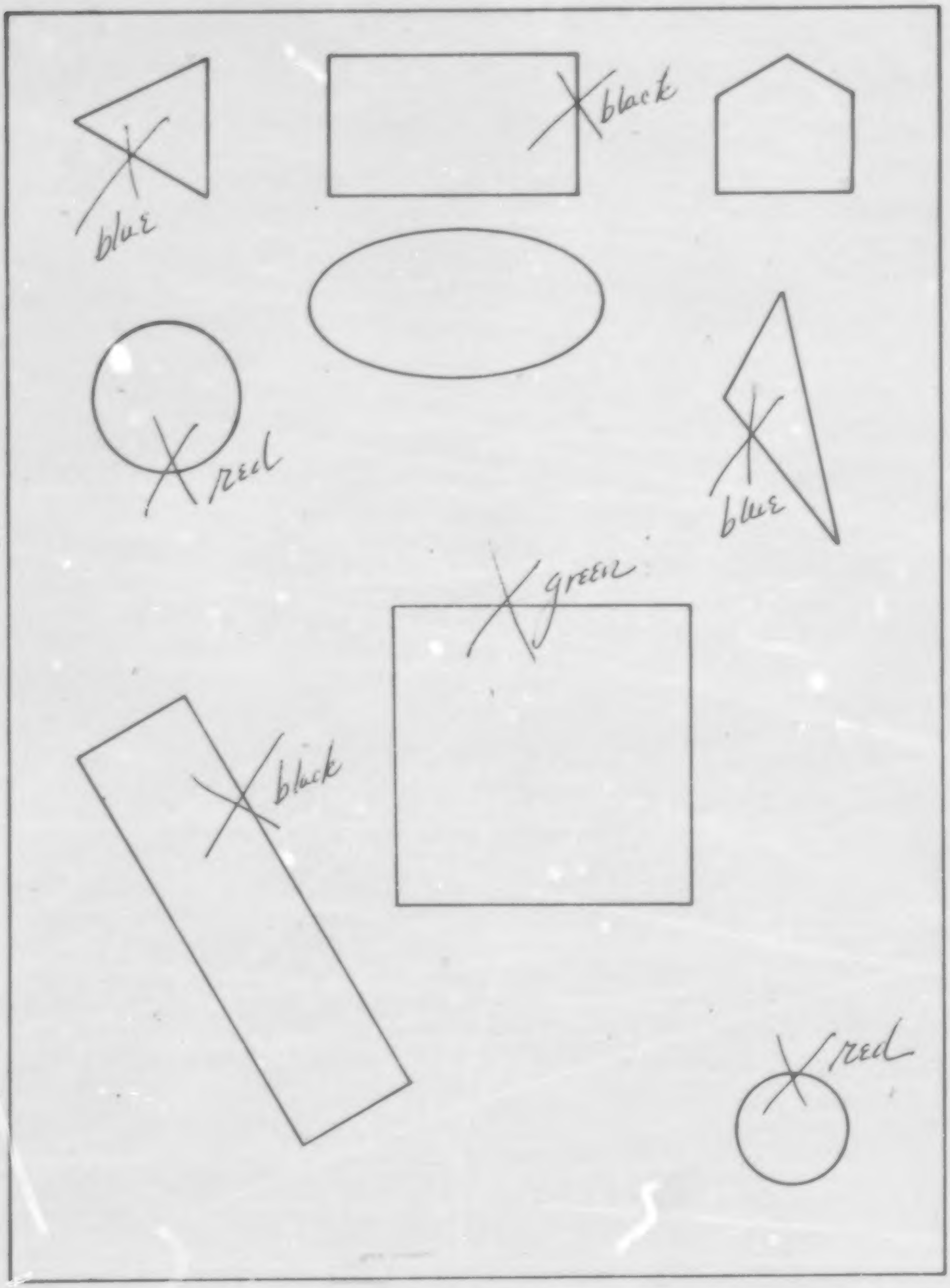
## Regions

Mark an X on each of the rectangles, squares, circles and triangles.


green
black
blue
red

## Regions

Mark an X on each of the rectangles, squares, circles, and triangles.


black
blue
red
blue
green
black
red

FURTHER ACTIVITIES:

1. Place parquetry blocks in a bag for a game of identifying figures. If blocks are not available, figures cut from tagboard or cardboard may be used. Children take turns. Each reaches into the bag without looking and identifies the shape of a block by feeling it. He may say, for example, "The block is shaped like a triangle." Then he brings out the block. If the other players agree that he is correct, he places the block in front of him. Otherwise he returns it to the bag. At the end of the game, the child having the most blocks is the winner. It is necessary, of course, to establish the rule that each child must have the same number of turns. Children can make tally marks to keep track of their turns.

2. Start Our Big Book of Shapes with a page for each of the figures--rectangular region, triangular region, and circular region. Paste a model cut from construction paper at the top of each page. Children may cut pictures from magazines and paste them on appropriate pages. Do not hastily reject a child's selection as incorrect; inquire. Some aspect or detail that escapes your attention may have been seen by the child.

3. Give children geometric regions cut from colored construction paper. They may assemble the shapes into "pictures" of animals, people, boats, buildings, tree forms, and so on.

4. Provide parquetry blocks and design blocks for children to use in making designs, pictures, etc. Further intuitive understanding among geometric figures can be developed by such experiences. The intent of this chapter is to introduce concepts and vocabulary rather than to have children "master" the content.

## 5-5. FITTING REGIONS

**OBJECTIVE:** To distinguish different regions by seeking to fit them on each other.

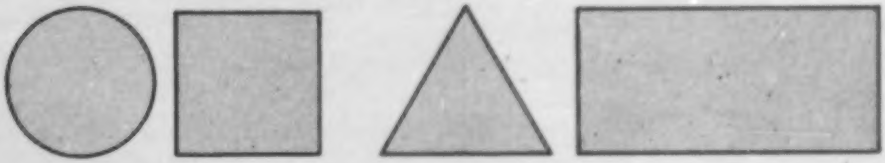
**VOCABULARY:** Match, fit.

**MATERIALS:** Flannel board regions of different sizes and shapes; there should be two sets of congruent figures of contrasting colors (red and green, for instance); one square region clearly larger than any of the other congruent figures; a few sets of construction paper regions in two colors, as above.

### SUGGESTED PROCEDURE

### PRE-BOOK ACTIVITIES:

Place on the flannel board some of the red figures as shown:

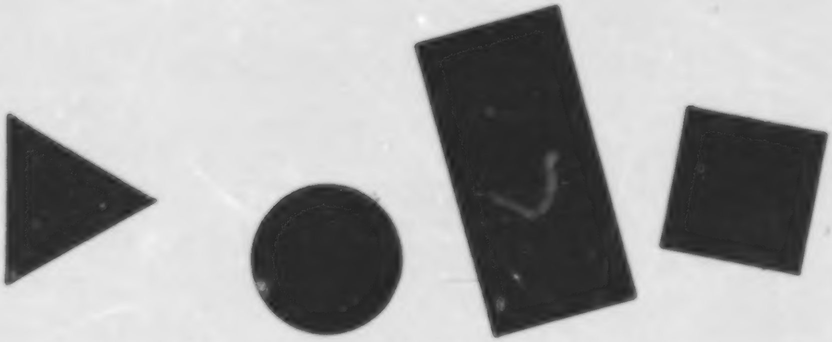


Talk about what it means to fit exactly, or to match exactly. Show how edges of coins of like denomination match exactly; discuss the way the edges of slices of bread often fit exactly in a loaf. Pages in a book match, and one end of an unsharpened pencil may fit exactly against the end of another, with nothing left over of either pencil.

Hold up a green rectangular region which will fit exactly one of the red ones on the flannel board. Hold it with its sides parallel to the sides of the one on the board. Have it described as a rectangular region. Ask whether this region will exactly fit any of those on the flannel board. Have a child do the matching, and show that all sides match, or fit.

Remove the two figures, place the green one on the flannel board, and ask whether the red one can be matched to it. Remove the green figure and ask whether it would fit on any of the other regions of the flannel board. Have a child try to fit it, and show clearly that there are some parts not covered up either on the red or on the green figure. Match the other green regions to the appropriate red regions in the same way, having them described each time as a _____ region.

Without letting the class see what you are doing, arrange the green regions on the flannel board in different positions.



Hold up the red triangular region in the same position it was in when it was first matched. Ask with what kind of region it might be matched. Turn the flannel board so that the children can see the different shapes; then ask the children whether the red region can be matched to one of those on the board. Caution the children to be careful, for even this simple arrangement can cause difficulty for children who expect to see regions in positions with one side parallel to the floor. Continue fitting the other figures.

Hold up the square region of larger size and ask whether it could be matched to any of those on the board. Discuss the fact that a region must not only be the same shape but also the same size in order to fit exactly.

## USING THE PUPIL'S BOOK, PAGES 147-150: REGIONS THAT FIT

### Ideas

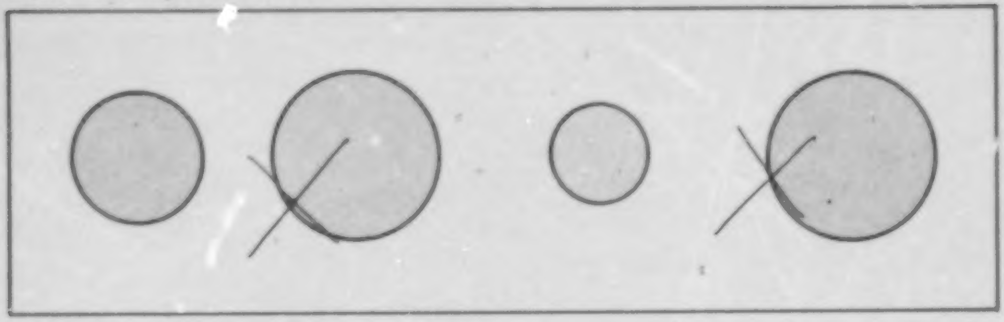
Regions of the same size and shape can be fitted one on the other.

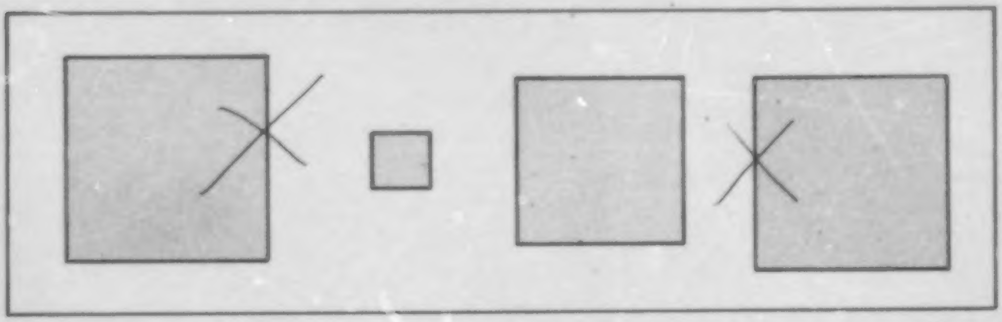
A region can be rotated for possible fitting.

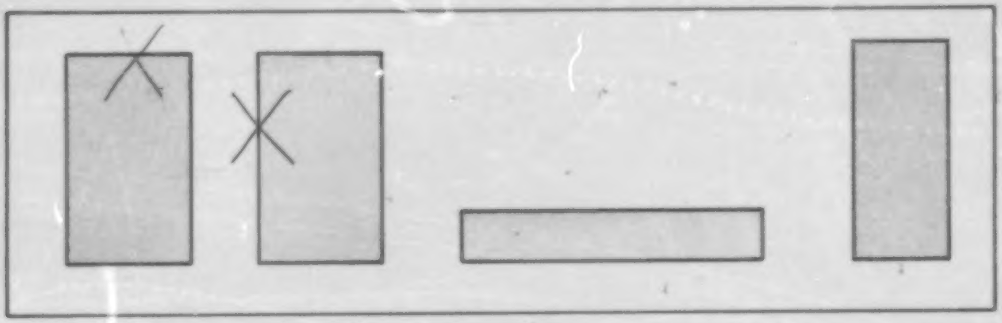
The first two pages show regions in the so-called horizontal position. It represents no problem for identification. The third and fourth pages will need more careful scrutinizing since rotating will be necessary in most instances to get the figures to fit.

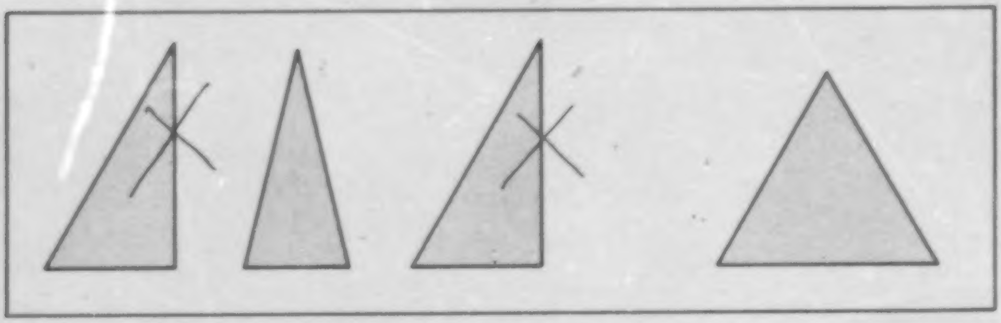
## Regions that Fit

Mark the regions that fit.



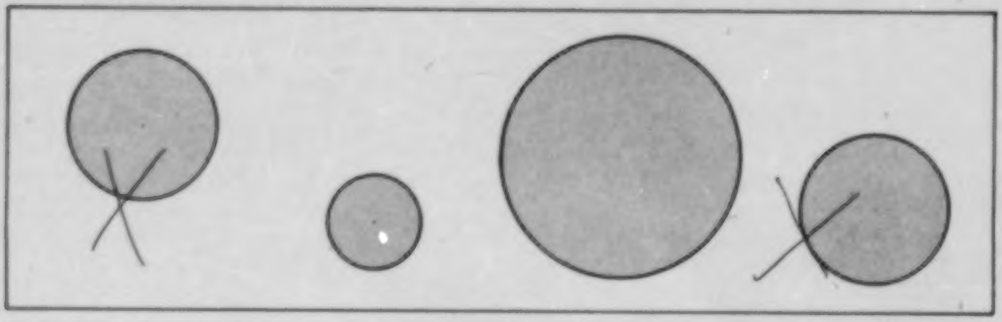


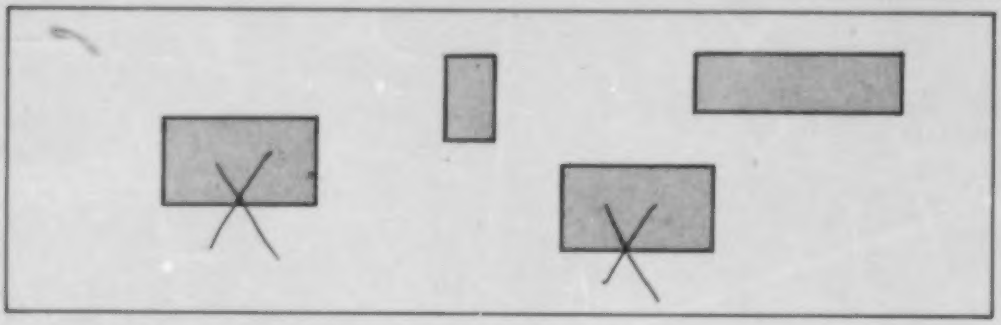


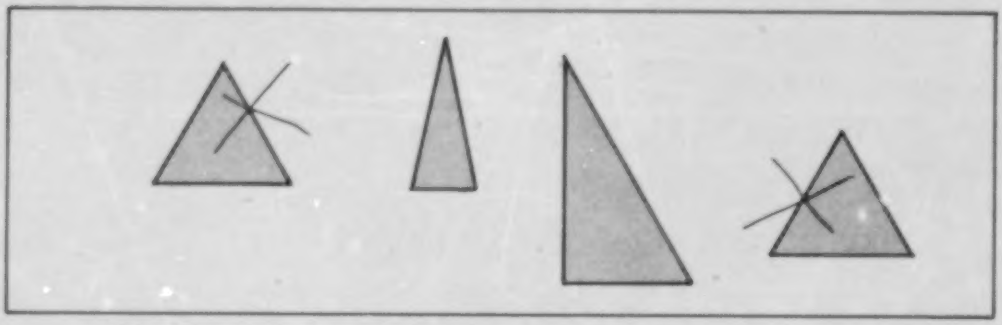


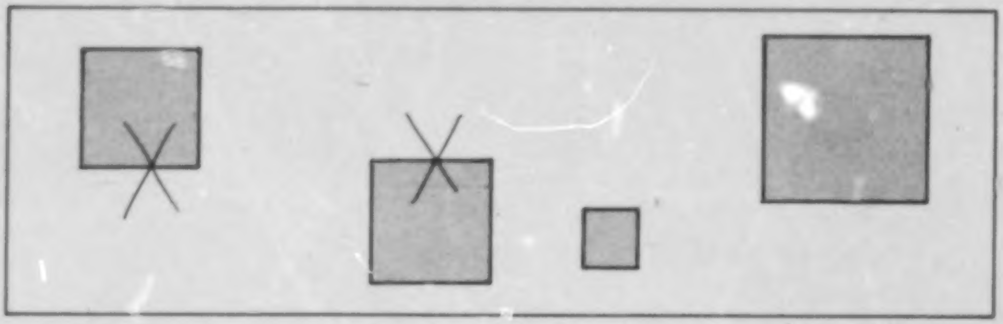
## Regions that Fit

Mark the regions that fit.



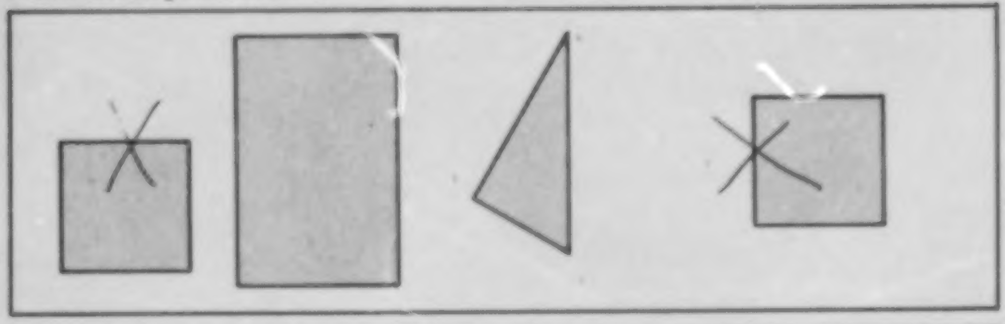


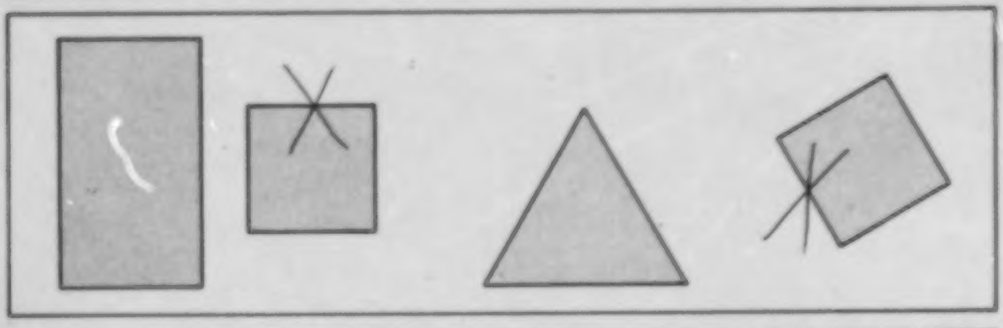


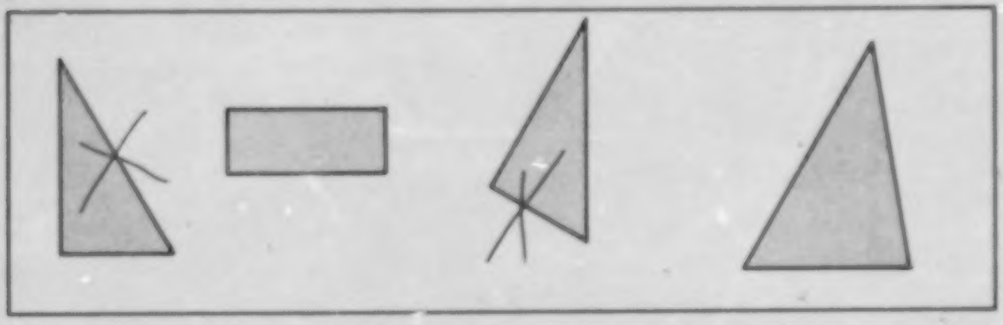


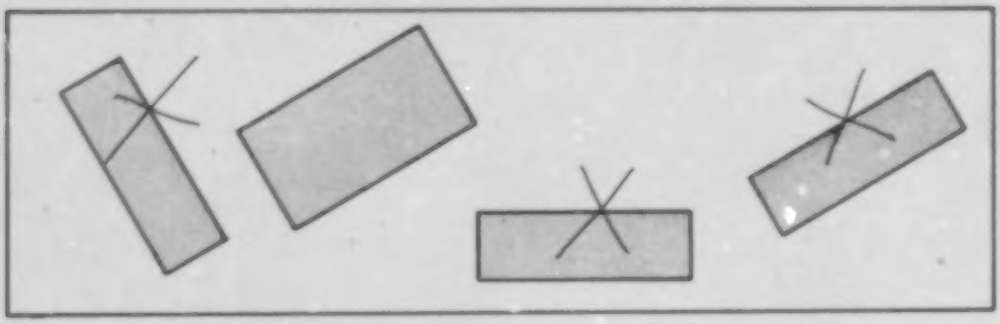
## Regions that Fit

Mark the regions that fit.



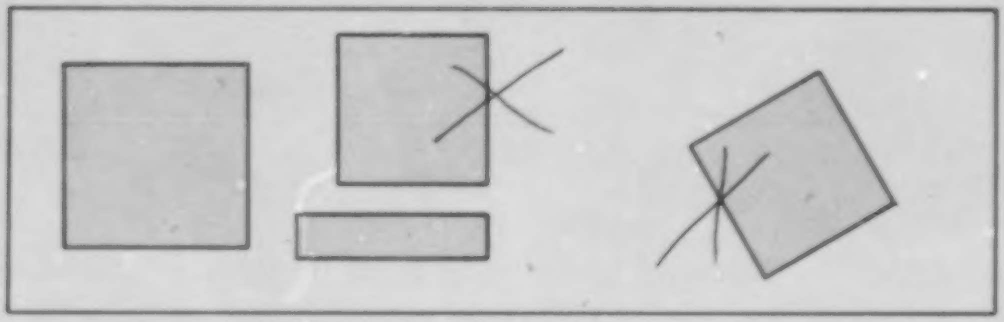


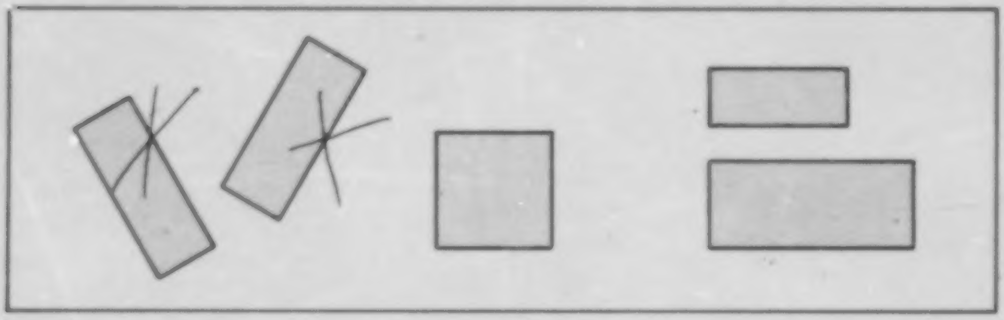


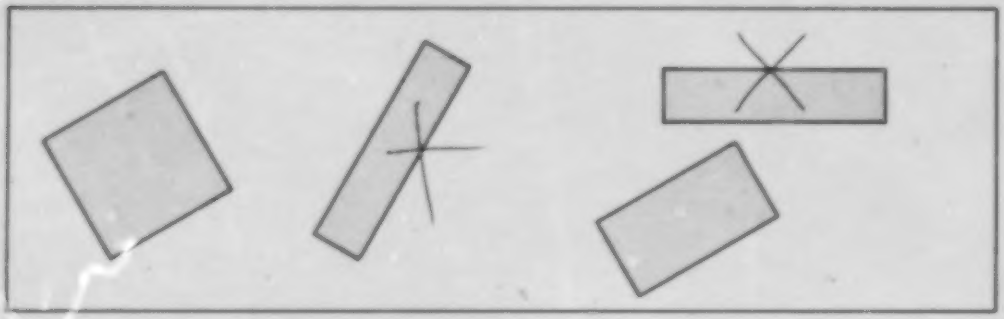


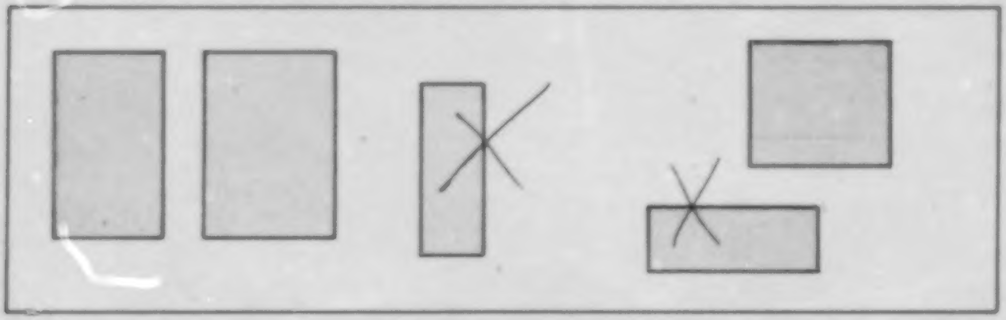
## Regions that Fit

Mark the regions that fit.



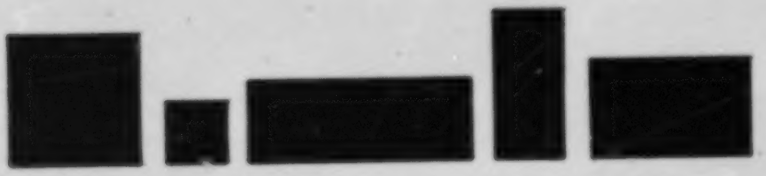




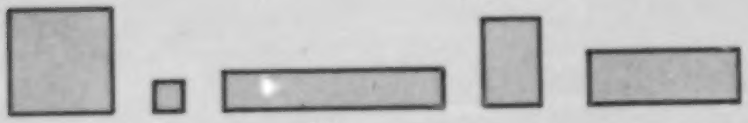


- An additional series of lessons can be developed to refine comparisons by fitting. A long thin rectangular region can be included with one that is nearly square.

All the red regions might be rectangular regions (including some square ones) such as two different sized square regions and three or more rectangular regions of different dimensions and proportions.



The green regions should include all of these shapes as well as other rectangular (and square) regions of different proportions, including some like the following for which one pair of sides fits somewhere above but the other does not.



This time, the pupil will need to recognize that, for the fitting, the lengths of opposite sides must be the same. Choose some of the green regions quite similar to the red ones, but not actually the same; good practice can then be developed in estimating relative lengths. In most cases it would be profitable, before any attempted fitting is made, to discuss whether a given green region will fit and what would be reasonable places to try it.

Another time, some of the red regions on the board should be turned in different positions. It is important to plan specifically for such a lesson.

Geometric insights of these additional lessons would include:

1. An awareness of the impossibility of matching a long, thin rectangle with one which is nearly a square;
2. An awareness of the possibility of comparing visually the sides of a region to see whether they are likely to fit;
3. An awareness of the possibility of rotating a region to make it fit another;
4. The recognition of a rectangular or triangular region which does not have a side parallel to the floor.

## Chapter 6

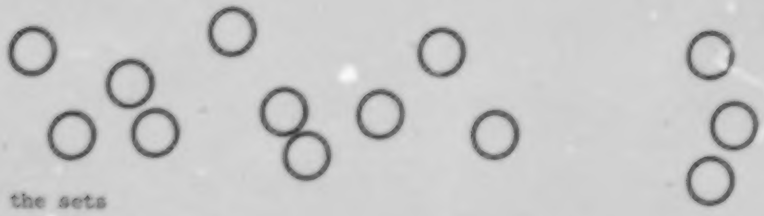
## PLACE VALUE AND NUMERATION

### Background

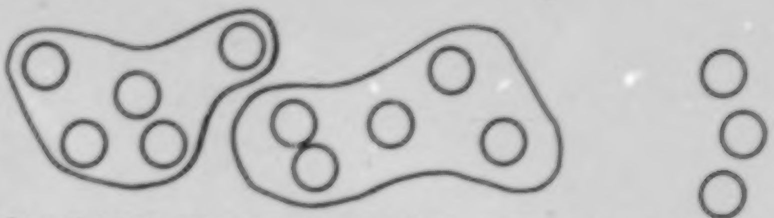
The fundamental purpose of this chapter is to learn assigned names of numbers greater than nine. We have named the first few numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10, but the procedure of assigning a new name to each successive number is clearly impractical. Some sort of system of naming numbers is necessary. This chapter is devoted to the *Hindu-Arabic* system of numeration, our *decimal system* of numeration. It is interesting to notice that this is a relatively modern system--quite unknown to the Greeks and Romans. Indeed, mathematicians have conjectured that the rather feeble accomplishment of the Greeks in algebra was due to their lack of a reasonable notational system. The system which we now use is only about a thousand years old; it was carried to Europe, along with spices and sandalwood, by Arab traders.

The simplest numeration systems are very closely related to tallying. For instance, the Romans used I, II, and III for the first three numbers. Of course, this sort of notation is completely impractical for large sets, and people soon found ways of simplifying the naming system. The first step was to count by groups of some agreed-upon size, so that, for example, we might refer to seven dozen eggs, or a gross (twelve dozens) of pencils.

Let us state in mathematical terminology just what this sort of "grouping" amounts to. Suppose we are trying to describe a set which has a great many members. We select a subset of some standard number of members (like a dozen, or a gross) and *partition* (split up) the set into as many equivalent subsets as possible. There may or may not be a *remainder* (that is, members left over). Thus if 5 is the standard number, we may partition the set



into the sets



and describe the original set as consisting of 2 fives and 3 ones. The number of members of the standard subset is more or less arbitrary. Thus we describe the number of members of the set pictured above in any of the following ways:

| Fives | Ones |
|---|---|
| 2 | 3 |

| Threes | Ones |
|---|---|
| 4 | 1 |

| Tens | Ones |
|---|---|
| 1 | 3 |

We customarily group by tens--presumably because we have ten fingers. Computing machines customarily group by twos, and the barefoot Mayans grouped by twenties.

This system of counting by groups has been used by most civilizations. But as greater and greater numbers of objects were considered, new names for greater and greater standard numbers became necessary. Thus the Romans used I, V, L, C, D, and M for one, five, fifty, one hundred, five hundred, and one thousand. At each stage, as names for greater numbers were needed, a new symbol was needed. But the Hindu system circumvents this difficulty by assigning meaning to the place a digit occupies, and manages to create numerals for every number from the ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. This is a truly a remarkable achievement.

The idea of grouping, together with place value, is enough to permit us to assign numerals to the first hundred numbers. The step from the pattern:

| Tens | Ones |
|---|---|
| 4 | 7 |

to the numeral 47

is a simple one, and it should be clear that this number is to be assigned to a set which consists of 4 tens and 7 ones. The number 10 is described in precisely the same way: this is the number which is to be assigned to a set of 1 ten and 0 ones. We say that the right hand digit is in the ones' place, and that its neighbor on the left is in the tens' place.

There is a further step in our system of numeration. Suppose that a set consists of 23 tens and 4 ones. In counting the 23 tens we would normally group these in tens, so that our record keeping might look like either of the following:

| Tens | Ones |
|---|---|
| 23 | 4 |

| Tens of tens | Tens | Ones |
|---|---|---|
| 2 | 3 | 4 |

In either case, naming this number 234 is completely natural. We say that the right hand digit is in the ones' place, its left hand neighbor in the tens' place, and the next left hand digit is in the hundreds' place. We call tens of tens "hundreds", and we call tens of tens of tens "thousands". But the practice of naming these greater numbers eventually becomes impractical and we fall back on the numerals. Thus

234,468,789,345,863,456,998,567,452,345,765,989

names a certain number in a perfectly well-defined way, but it is doubtful if many of us remember the ordinary names beyond quadrillion.

NOTE:

The sequence of topics in this chapter may require a little explanation. We begin by partitioning a set into as many sets of ten as possible. We then record the number of sets of ten (number of tens) and the number in the remaining set (the number of ones). Then we begin to name these numbers. Following this introduction to the meaning and writing of 2-digit numerals, the so-called "teen numbers", 11 through 19, are considered.

Difficulties arise because the pattern of naming is more complex than the naming of the numbers in the twenties, thirties and so on. Eleven and twelve have very special names, but the names of the "teens" reverse the usual pattern. For instance, in "thirteen" the first part of the word is associated with ones and the last part with ten. On the other hand, in "twenty-seven" the reverse is true.

Again, note that the numeral "10" was delayed until we could assign it the natural meaning: one ten and zero ones.

## 6-1. COUNTING BY TENS AND ONES

OBJECTIVE: To help children learn to count sets with many members by counting sets of ten.

VOCABULARY: (No new terms.)

MATERIALS: Flannel board squares and strips of ten, similar flannel board material, other types of counting materials.

BACKGROUND NOTE:

A set of objects may be partitioned into subsets of ten members each and a set of not more than 9 objects. (We do not use the term "partition" with the children.) In this lesson the children learn to do this partitioning into subsets of ten and to name the number of members in the set; e.g., 3 tens and 7 ones, or (orally) thirty and seven.

TEACHING NOTE:

There are very few pages in the pupils' book for use with this lesson. This is not an oversight. Teachers have found that actual manipulation of sets of objects is much more effective than working with pictures of sets. Such pictures necessarily either group the members of the set artifically or else present an impossibly cluttered appearance.

### SUGGESTED PROCEDURE

PRE-BOOK ACTIVITIES:

Place thirty-four flannel cut-outs in a box. Ask the children to count with you as you remove ten from the set and place the objects in a row on the flannel board. Be sure they understand that "ten" or "ten ones" are the names that tell how many. Place another set on the flannel board that matches the set already there.

How many members in the second row? (Ten.)
Do we have to count to know that there are ten? (No. The first row has ten members and the second row matches the first.)
How many sets of ten are on the flannel board? (2.)

Do the same for a third set of ten. Then show the remaining 4 objects.

Do we have enough to make another row of ten? (No.)
How many sets of ten do we have? (3.)
What is another name for three sets of ten? (Thirty.)

What number tells how many objects are not in sets of ten? (4.)

These are the ones.

How many ones are there? (4.)

How many objects were in the box? (Many answers should be given, such as 3 tens and 4 ones, thirty plus four, thirty-four.)

If we counted the members of the set of the flannel board, are you certain that there would be thirty-four members? (Yes. Three tens and 4 ones are thirty-four.)

Just to be sure, let's count together.

The counting, of course, verified that 3 tens and 4 ones are the same as thirty-four ones. Counting is introduced here to emphasize this concept and to provide practice in counting beyond ten.

Repeat the experience with a set in which the number of members is 40.

Now how many sets of ten do we have? (4.)

We have separated all our material into sets of tens.

Do we have a set of ones? (No. There are no members in the set of ones.)

What is the number that we use to tell that a set has no members? (0.)

How many sets of ten do we have, and how many ones? (4 tens and 0 ones.)

Use other types of material to develop understanding of counting by tens and ones.

Provide each child with sets of paste sticks or other small objects to use in counting sets of tens and ones. It is not necessary to count these set materials before giving them to the children. For example, if paste sticks are used, just give each child a good handful. You will want a random assortment of number names. Since some children are more proficient in counting and/or pairing, wait until everyone has counted at least two sets of ten. Then collect the uncounted paste sticks. Be sure to leave the majority of children with some remaining sets of ones. You will want the recording of the numerals for these numbers to vary from 0 - 9 in the one's column. Draw a chart on the chalk board to use in recording the names of the numbers. Explain the meaning of the words. Then call on the children to tell how many tens and ones they have counted. As each child answers, ask where the numerals for the numbers that tell how many tens and how many ones should be written. Emphasize that the ten's numeral is written first; then the ones. Use every opportunity that presents itself

| Tens | Ones |
|---|---|

to ask about the placement of the ten's and one's digits as well as the name of the number until you are certain that every child understands (probably several weeks!). As each numeral is written ask for the name of the number, e.g., twenty-five, sixty-one, etc.

Put sets of small objects for counting into boxes or envelopes. Write a letter of the alphabet on each box or envelope. Give each child a paper which is marked:

A ___tens ___ones

B ___tens ___ones

C ___tens ___ones

D ___tens ___ones

E ___tens ___ones

Children may work as teams or alone to count contents of envelope and record the number of sets of tens and ones. After a child has completed one envelope, he replaces its contents and exchanges it for another envelope.

One child of a team may serve as the recorder or each may want to keep his own chart. A class chart can be used in order to verify the independent charts. Children can help in setting up materials of this kind. The number of objects in the envelopes can be changed and the activity repeated.

### USING THE PUPIL'S BOOK, pages 151-152

Ask the children to ring sets of ten, count the number of sets they have ringed, and record the numeral in the ten's column. Then determine the number of ones and record this numeral in the one's column. No doubt you will want to provide additional work sheets of this kind for children who need to develop more skill in performing this kind of task.

### FURTHER ACTIVITIES:

1. There is a decided advantage when teaching the "teens" numbers if children already know the names of these numbers. Prepare for this by providing some daily experience in counting children, chairs, books, days on the calendar, etc. This does not mean that the children should not be permitted to count beyond 20. Increase the number of objects to be counted as the children master the names of the numbers.

2. Repeat the procedure of having children count sets of tens and ones as many times as is necessary to develop the understandings.

3. Provide a number line with the whole numbers from 0 through 100 that can be attached to the sides of the chalk board in such a manner that children can touch the numerals. Hold up ten bundles of ten and ask one child to point to the numerals on the number line as the children count the sets by tens. A great deal of practice will be needed before most children will be able to locate the numerals for the multiples of ten as quickly as the children name the numbers.

## Tens and Ones

XXXXXXXXXX
XXXXXXXXXX
XX

| Tens | Ones |
|---|---|
| 2 | 2 |

XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XX

| Tens | Ones |
|---|---|
| 3 | 2 |

XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXX

| Tens | Ones |
|---|---|
| 3 | 5 |

XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX

| Tens | Ones |
|---|---|
| 4 | 0 |

XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
X

| Tens | Ones |
|---|---|
| 4 | 1 |

XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
X

| Tens | Ones |
|---|---|
| 3 | 1 |

XXXXXXXXXX
XXXXXXXXXX
XXXXXX

| Tens | Ones |
|---|---|
| 2 | 6 |

XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXX

| Tens | Ones |
|---|---|
| 4 | 9 |

## Tens and Ones

XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXX

| Tens | Ones |
|---|---|
| 3 | 5 |

XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX

| Tens | Ones |
|---|---|
| 5 | 0 |

XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
X

| Tens | Ones |
|---|---|
| 5 | 1 |

XX

| Tens | Ones |
|---|---|
|  | 2 |

XXXXXXXXXX
XXXXXXXXXX
XXX

| Tens | Ones |
|---|---|
| 2 | 3 |

XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX

| Tens | Ones |
|---|---|
| 7 | 0 |

XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXX

| Tens | Ones |
|---|---|
| 3 | 4 |

XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XXXXXXXXXX
XX

| Tens | Ones |
|---|---|
| 5 | 2 |

## 6-2. ELEVEN, TWELVE, AND THE TEENS

OBJECTIVE: To associate the spoken names and written numerals for the numbers 11-19.

VOCABULARY: Eleven, twelve, thirteen, ... nineteen.

MATERIALS: Flannel board materials, blocks, sticks, etc. (One ten and ten ones of each.)

### SUGGESTED PROCEDURE

### PRE-BOOK ACTIVITIES:

Place ten objects on the flannel board and have children name the number of the set. Refer to the ten's and one's chart drawn on the chalkboard.

How many sets of ten? (One.)
Where should I write the numeral in the chart? (In the first column.)
Did we have any members left over after we made the set of ten? (No.)
What numeral should I write in the one's column? (Zero.)

Place another object on the flannel board, underneath at the left.

How many do we have now? (One ten and one, or ten and one more.)

If someone suggests that there are eleven, ask the children to go back to the beginning and count by ones to determine that eleven is the correct name. If no one suggests eleven, supply the word and then ask the children to count.

How many sets of ten? (1.)
Where should I write the numeral that names the number of tens? (In the ten's column or in the first column.)
How many ones? (1.)
Where should I write this numeral? (In the one's column or in the second column.)
Look at the numeral (11) carefully.
Do both of these ones mean the same thing? (No. One means one ten and the other one, one.)
Which one means one ten? (The one in the first column. The one in front.)

Using the same procedure, place another object and develop the idea of twelve as 10 and 2 more, or twelve ones. Continue with thirteen. Be sure children know they are saying thirteen, not thirty. This same warning applies to the other teen names which are closely related to the names of the multiples of ten. Be careful to enunciate very carefully so that children will hear the difference. (Notice that if the names for sets with one ten and some ones followed the same pattern as the other number names of the twenties, thirties, etc., we would say something like "onety-one, onety-two, etc." It would not be quite so hard if we said, "Teen-one, teen-two, teen-three, teen-four", but that just isn't the way it's done in the English language!) Use other sets of materials and emphasize the oral names, and the idea of one ten and so many ones. List the numerals vertically.

Discuss how the numerals for ten, eleven, twelve and thirteen are alike (1 in ten's column) and different (number of ones increases by one so pattern is 0, 1, 2, 3). With some classes, it probably will be wise to stop at "thirteen" the first day. The number of teens introduced per day will depend upon the teacher's judgment of the understanding of the children. It would be desirable to return to 10 and review each time new numerals are introduced. A thorough understanding of the teens will save much difficulty later on.

- Many children will benefit from practice in writing the numerals 0 - 19 in both a vertical and a horizontal form, e.g.,

| | |
|---|---|
| 0 | 10 |
| 1 | 11 |
| 2 | 12 |
| etc. | |

and

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | etc. |
| 10 | 11 | 12 | |

It is helpful to provide worksheets that have been ruled to indicate the arrangement desired.

USING THE PUPIL'S BOOK, page 153

Ask children to ring sets of ten and record the numeral in the ten's column. Then determine the number of ones and record this numeral in the one's column.

Pages 154 - 155:

For each set, make a ring around the numeral that names the number of members in the set.

## Tens and Ones

XXXXXXXXXX

| Tens | Ones |
|---|---|
| 1 | 0 |

XXXXXXXXXX
X

| Tens | Ones |
|---|---|
| 1 | 1 |

XXXXXXXXXX
XX

| Tens | Ones |
|---|---|
| 1 | 2 |

XXXXXXXXXX
XXX

| Tens | Ones |
|---|---|
| 1 | 3 |

XXXXXXXXXX
XXXX

| Tens | Ones |
|---|---|
| 1 | 4 |

XXXXXXXXXX
XXXXX

| Tens | Ones |
|---|---|
| 1 | 5 |

XXXXXXXXXX
XXXXXX

| Tens | Ones |
|---|---|
| 1 | 6 |

XXXXXXXXXX
XXXXXXX

| Tens | Ones |
|---|---|
| 1 | 7 |

XXXXXXXXXX
XXXXXXXX

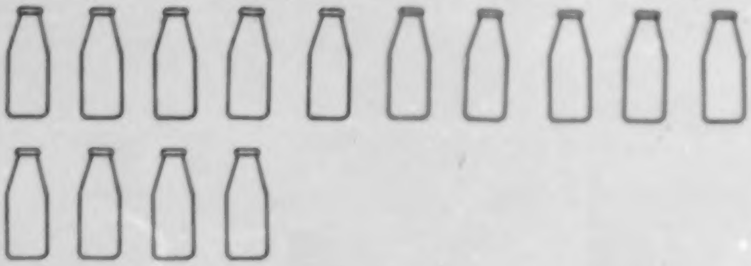
| Tens | Ones |
|---|---|
| 1 | 8 |

XXXXXXXXXX
XXXXXXXXX

| Tens | Ones |
|---|---|
| 1 | 9 |

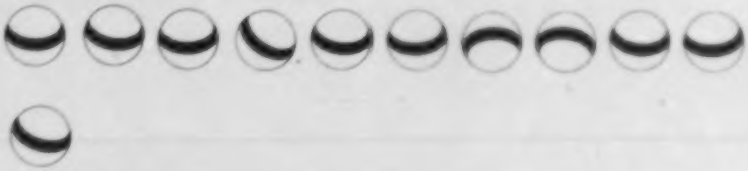
300

How many?

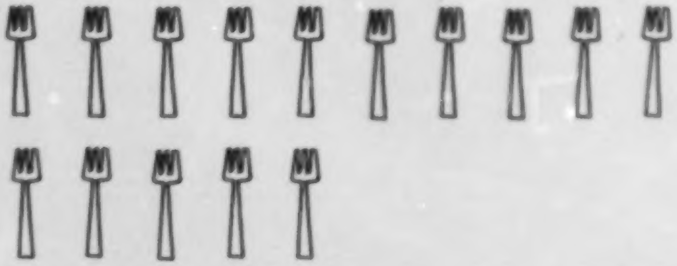


[✓ 14]
11
17

13
[✓ 17]
19

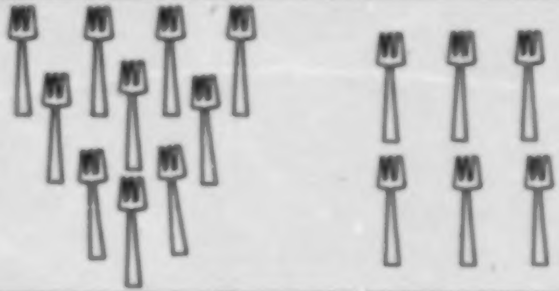


14
[✓ 11]
16

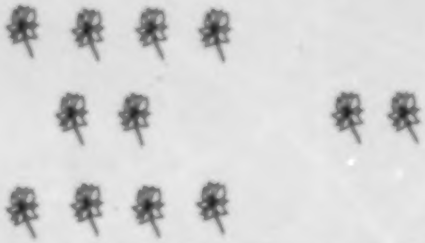


18
19
[✓ 15]

How many?

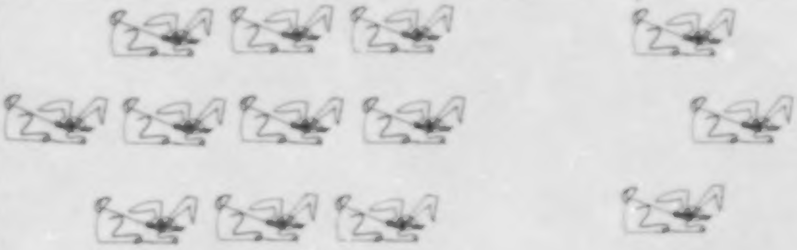


[✓ 16]
12
14



[✓ 12]
15
17

18
14
[✓ 19]



[✓ 13]
17
10

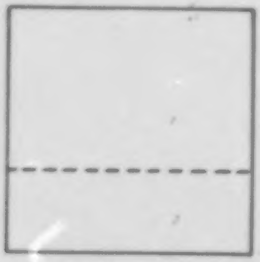
## FURTHER ACTIVITIES:

Give many opportunities for separating sets fewer than 20 into tens and ones, recording and saying the result as tens and ones and as the teen number.

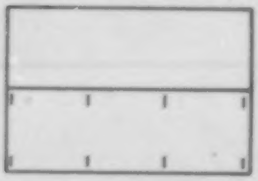
1. Hold up a numeral card and ask the children to show a set with the appropriate number of tens and ones.
2. Write numerals on chalkboard and have the children draw a ring around the digit that names the tens or the digit that names the ones.
3. With "SHOW-ME" cards have children show the numeral from ten-and-ones instructions and from the spoken word. (See below for construction of "SHOW-ME" cards.)

### HOW TO MAKE "SHOW-ME" CARDS

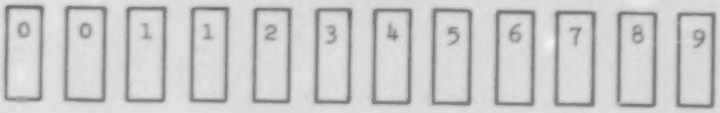
1. Use a piece of tagboard 6" x 6". Fold up 2" from the bottom.



2. Staple as marked to make 3 pockets, each almost 2" wide.



3. Cut a strip of tagboard 18" x 4" into 12 strips $1\frac{1}{2}$" x 4". With felt pen, write numerals as follows



4. Children should be taught early to lay out numeral cards in order on their desks and to replace them in order.

5. In the game, the children figure out the solution to a problem you give orally or on the chalkboard. They then place the numerals for the answer in the pockets, hold the cards against their chests with the answers concealed until you say, "Show-me!" Then all turn the answers toward you, while you make a quick survey to see who is right.

Note:

Show-Me cards and cards on which the numerals may be written can be purchased reasonably at any school supply house.

## 6-3. SPOKEN NAMES OF THE NUMBERS: 21 THROUGH 99

OBJECTIVE: To develop the ability to name the cardinal number of a set of more than twenty members by identifying a multiple of ten and counting on from that number.

VOCABULARY: The spoken names of numbers from 21 through 99.

MATERIALS: Different kinds of objects in sets of ten and ones.

TEACHING NOTE:

Children who are not proficient in counting sets of ten (ten, twenty, thirty, etc.) will find counting on from a given multiple difficult. Work sheets for pupils are not recommended here.

SUGGESTED PROCEDURE:

Place 2 tens and 5 ones on the flannel board.

> Who can tell how many members are in the set on the flannel board? (Two tens and 5 ones, or possibly twenty and five.)

Cover the 5 ones with tagboard.

> How can we count the tens in a short way? (Ten, twenty.)
>
> Then let's go on from twenty: twenty-one, twenty-two, twenty-three, twenty-four, twenty-five.

Repeat with several different sets. Children will need a great deal of practice counting together in this way.

- Prepare materials in sets of ten and ones. Give each child two bundles of ten and nine ones. Ask the children to represent the numbers one through nine, then ten. Since the children were not given 10 ones, the only representation of ten available to them is one ten. Watch to see that every child clears the top of the desk and displays only one set of ten. Continue with the representation of 11- 19. As each number is represented, ask the children to count the members to make certain the models are correct. This will reinforce counting on from a given multiple of ten. Then emphasize that the next number is twenty, which can be represented by two bundles of ten. Continue on through 29. A lesson like this should be repeated several times with these and other decades.

Children who can represent with set material the number that is one greater than, say, 39 by showing 4 sets of ten display that they have learned something about the base of our number system.

Note:

While working with bundles of tens and ones, you are teaching children about the base of our number system. Do not confuse this with place-value. Place-value is involved only if you represent the number of these bundles of tens and ones in numeral form, e.g., 39, 47, etc. A chart may be used in teaching place-value if you use the digits as in: You can also use strips of some sort that do not differentiate between tens and ones unless placed in the chart as in for 32.

| Tens | Ones |
|---|---|
| 3 | 2 |

| Tens | Ones |
|---|---|
| \|\|\| | \|\| |

## 6-4. THE WRITTEN NUMERALS: 20 THROUGH 99

**OBJECTIVE:** To help children associate the correct written numerals, as well as spoken names, with the numbers 20 to 99.

**VOCABULARY:** (No new words.)

**MATERIALS:** Different kinds of objects for counting: blocks, sticks, pegs, flannel board materials, etc. Show-Me cards for further activities.

### SUGGESTED PROCEDURE

### PRE-BOOK ACTIVITIES:

This activity should follow many experiences with counting and naming sets of ten and single objects. The children should be able, for example, to name a set of 4 tens and 5 ones as forty and five, and as forty-five.

Place three stacks of ten blocks and a stack of two blocks on the chalk tray. Have children name, in several ways, the number of members in the set. (3 tens and 2 ones; thirty-two.)

Begin a tabulation on a chart showing:

| Tens | Ones |
|---|---|
| 3 | 2 |

Place 5 sets of ten small objects on the flannel board. Have children tell, in two ways, how many there are. Make 2 bundles of ten sticks each and put them with 6 sticks on a table. Ask a child to tell the number of tens and ones. Continue to develop the chart as each of these sets is counted. Show 4 sets of ten and 7 ones. Have children tell where you should write the numeral for the tens and the numeral for the ones for each set of objects.

The chart might look like this.

| Tens | Ones |
|---|---|
| 3 | 2 |
| 5 | 0 |
| 2 | 6 |
| 4 | 7 |

Each of these numbers can be named in several different ways.

When we see the numerals on the chart, we read 3 tens and 2 ones.

We may say thirty and two or thirty plus two.

We also say thirty-two.

We write: 32.

Continue to rewrite the numerals from the chart.

A completed chart might look like this:

| Tens | Ones | Numerals |
|---|---|---|
| 3 | 2 | 32 |
| 5 | 0 | 50 |
| 2 | 6 | 26 |
| 4 | 7 | 47 |

USING THE PUPIL'S BOOK, page 156:

In each case the child is to show the other way to name the number.

Page 157:

Write the numeral which names the number of objects in the set.

# Two Names For A Number

Write another name.

| Tens | Ones | | |
|---|---|---|---|
| 2 | 7 | is | 27 |
| 4 | 7 | is | 47 |
| 3 | 0 | is | 30 |
| 5 | 6 | is | 56 |
| 9 | 6 | is | 96 |
| 8 | 3 | is | 83 |
| 2 | 9 | is | 29 |

How many?

| | | | |
|---|---|---|---|
| XXXXXXXXXX<br>XXXXXXXXXX<br>XXXXXXXXXX<br>XXXXXXXXXX | 40 | 0000000000<br>0000000000<br>0000000000<br>0000000000<br>0000000 | 47 |
| FFFFFFFFFF<br>FFFFFFFFFF<br>FFFFFFFFFF<br>FFFFFFFFFF<br>FFFFFFFFFF<br>FF | 52 | PPPPPPPPPP<br>PPPPPPPPPP<br>PPPPPPPPPP<br>PPPPPPPPPP<br>PPPPPPPPPP | 50 |
| * * * * * * * * * *<br>* * * * * * * * * *<br>* * * * * * * * * *<br>* * * * * | 35 | ¢¢¢¢¢¢¢¢¢¢<br>¢¢¢¢¢¢¢¢ | 18 |
| ++++++++++<br>++++++++++<br>+ | 21 | &&&&&&&&&&<br>&&&&&&&&&&<br>&&&&&&&&&&<br>&&&&&&&&&&<br>&&&&&&&& | 48 |

## 6-5. ORDER RELATIONS FOR NUMBERS 0 THROUGH 99

**OBJECTIVE:** To extend the ideas of "greater than" and "less than" to include the numbers 11 through 99.

**VOCABULARY:** (Review) greater than, less than.

**MATERIALS:** Sets of sticks or small counting materials bundled into tens and ones; a number line.

### SUGGESTED PROCEDURE

### PRE-BOOK ACTIVITIES:

Place a set of 3 tens and a set of 4 tens on a table or desk. Identify the members of the sets as sticks in bundles of ten and ask the children which set has more members. This may be determined by counting by tens and by pairing. Hold up the 3 sets of ten.

> *How many tens?* (3.)
> *What is a name for three tens?* (Thirty.)
> *What numeral names thirty?* (30.)

Write 30 on the chalkboard and repeat the same procedure with the 4 sets of ten. Record 40 under 30 on the chalkboard.

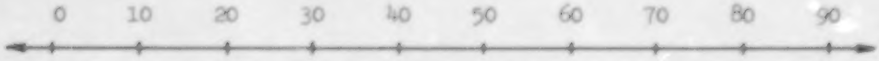
> *Which is greater, 30 or 40?* (Forty.)
> *How do you know this is true?* (A set of 4 tens or 40 has more members than a set of 3 tens or 30.)
> *Forty is how much greater than 30?* (One ten or 10.)

Leave these numerals on the chalkboard and repeat the procedure using 20 and 30, 40 and 50, etc.

> *Look at the numerals on the chalkboard.*
> *The first pair is 30 and 40.*
> *Which numeral names the greater number?* (40.)
> *How can you tell just by looking at these two numerals that this is so?* (Look at the ten's digit. 4 is greater than 3 so 4 tens is greater than 3 tens or 40 is greater than 30.)
>
> *Forty is how much greater than 30?* (One ten or 10.)
> *Can you tell from the numerals that 40 is 10 greater than 30?* (Look at the ten's digit. 4 is 1 greater than 3 so 4 tens is 1 ten greater than 3 tens. 40 is 10 greater than 30.)

Consider the other pairs of numerals in the same way. Help children generalize that if there are the same number of ones in two numbers, then we need only compare the tens.

- It is helpful to draw a number line like the one below on the chalkboard.



For example, place a finger under 30 and another under 40 and ask which numeral names the greater number. Encourage children to indicate how the arrangement of the numerals on the number line helps them determine the greater number. If a child says that 40 is greater than 30 because 40 is after or behind 30 on the number line, comment, "Yes, 40 is greater than 30 since 40 is to the right of 30 on the number line."

USING THE PUPIL'S BOOK, page 158:

Ring the numeral for the greater of the two numbers in each set.

Page 159:

Read the title. Discuss the first exercise with the pupils. Ask which of the numerals name numbers greater than 40. (50, 70). Direct the children to ring the numerals 50 and 70. Have the children work independently on the other exercises.

- Repeat the same procedure but ask for the number that is less than a given one and how much less. Do not compare numbers that differ by more than 10.

Which is greater?

| | | |
|---|---|---|
| 10<br>[✓ 20] | [✓ 30]<br>20 | 30<br>[✓ 40] |
| [✓ 90]<br>80 | 60<br>[✓ 70] | [✓ 50]<br>40 |
| 10<br>[✓ 50] | [✓ 90]<br>20 | 30<br>[✓ 60] |
| [✓ 70]<br>50 | [✓ 60]<br>20 | 50<br>[✓ 70] |

Which is greater?

| | | | | |
|---|---|---|---|---|
| 40 | 10 | [✓ 50] | [✓ 70] | 20 |
| 70 | [✓ 80] | 20 | [✓ 90] | 30 |
| 20 | [✓ 40] | 10 | [✓ 60] | [✓ 70] |
| 50 | [✓ 70] | 30 | [✓ 90] | [✓ 60] |
| 30 | 10 | [✓ 50] | [✓ 80] | [✓ 40] |
| 80 | 20 | [✓ 90] | 30 | 10 |

USING THE PUPIL'S BOOK, page 160:

Read the title. Direct the children to look at the first exercise. Ask which of the numerals names the lesser of the two numbers in the set. (70.) Direct the children to ring the numeral 70. Have the children work independently on the other exercises.

Page 161:

Read the title. Direct the children's attention to the first exercise. Ask the children to name two numbers named in the exercise that are less than 40. (20, 30.) Direct the children to ring the numerals 20 and 30. Have the children work independently on the other exercises.

Which is less?

| | | |
|---|---|---|
| 80<br>[✓ 70] | [✓ 20]<br>30 | 60<br>[✓ 50] |
| 40<br>[✓ 30] | [✓ 10]<br>20 | [✓ 60]<br>70 |
| 90<br>[✓ 10] | [✓ 60]<br>80 | [✓ 20]<br>40 |
| 50<br>[✓ 20] | [✓ 20]<br>90 | [✓ 60]<br>80 |

Which is less?

| | | | | |
|---|---|---|---|---|
| 40 | 50 | [✓ 30] | 80 | [✓ 20] |
| 70 | [✓ 60] | 90 | [✓ 40] | [✓ 50] |
| 50 | 70 | [✓ 20] | [✓ 40] | 80 |
| 90 | [✓ 40] | [✓ 60] | [✓ 10] | [✓ 30] |
| 60 | [✓ 30] | 80 | [✓ 40] | 70 |
| 20 | 60 | 90 | 70 | [✓ 10] |

Page 162:

This is a teaching page. Read the title page. Read the first exercise. When the children give the correct answer (20) select a child to write the numeral in the space provided. Direct the children to write the numeral in their texts. Continue for all the exercises. If a child has trouble let him use objects (sets of 10) to discover the answer.

Teaching Page

Name the number.

10 greater than

10 is 20

50 is 60

30 is 40

80 is 90

60 is 70

20 is 30

10 less than

20 is 10

90 is 80

40 is 30

50 is 40

70 is 60

30 is 20

- Place a set of 34 (3 bundles of ten and 4 ones) and a set of 64 (6 bundles of ten and 4 ones) on a table or desk. Ask the children which set has more members. This may be determined by counting (tens and ones) and by pairing. Point to the set of 34.

  How many tens? (3.)
  How many ones? (4.)
  What is the name of the number? (Thirty-four.)

Write 34 on the chalkbaord and repeat the same procedure with the set of 6 tens and 4 ones and write 64 under 34 on the chalkboard.

  Which is greater, 34 or 64? (64.)
  How do you know this is true? (A set of 6 tens and 4 ones or 64 has more members than a set of 3 tens and 4 ones or 34.)

Leave these numerals on the chalkboard and repeat the procedure using 42 and 62, 76 and 26, etc.

  Look at the numerals on the chalkboard.
  The first pair is 34 and 64.
  Which numeral names the greater number? (64.)
  How can you tell just by looking at these two numerals that this is so? (The one's digits are the same. Look at the ten's digit. 6 is greater than 3 so 6 tens is greater than 3 tens. 64 is greater than 34.)

Consider the other pairs of numerals in the same way. Help children generalize that since there are the same number of ones in the two numerals, we need only compare the tens as we did with the multiples of 10. The number line used with the multiples of ten can be used as an aid if the children realize that they need only compare the ten's digits.

USING THE PUPIL'S BOOK, page 163:

Read the title. Direct the children to look at the first exercise. Ask, "Which is greater 65 or 25?" When a child gives the correct answer (65) direct the children to ring the numeral 65. Children work independently on the other exercises.

Page 164:

Read the title. Direct the children to look at the first exercise. Have the children read each number named. Ask, "Which numbers named are greater than 33?" (83, 53, 73, 43; all of them.) Direct the children to ring numerals that name numbers greater than 33. Children work independently on other exercises.

320

Which is greater?

[✓ 65]
25

23
[✓ 43]

72
[✓ 82]

[✓ 42]
22

87
[✓ 97]

[✓ 56]
26

55
[✓ 95]

[✓ 66]
36

49
[✓ 69]

29
[✓ 49]

[✓ 99]
79

[✓ 43]
33

Which is greater?

| | | | | |
|---|---|---|---|---|
| 33 | [✓ 83] | [✓ 53] | [✓ 73] | [✓ 43] |
| 75 | [✓ 85] | 45 | [✓ 95] | 65 |
| 42 | 22 | [✓ 52] | [✓ 72] | 32 |
| 81 | 61 | [✓ 91] | 51 | 21 |
| 67 | 47 | [✓ 97] | 37 | [✓ 87] |
| 98 | 28 | 88 | 48 | 78 |

- Repeat the same procedure and ask for the number that is less than a given one.

USING THE PUPIL'S BOOK, pages 165 - 166:

Use the same procedure as you used on pages 163 and 164 for greater than.

Which is less?

[✓ 52]
82

[✓ 34]
54

76
[✓ 46]

93
[✓ 83]

[✓ 65]
85

87
[✓ 27]

[✓ 41]
61

[✓ 24]
54

99
[✓ 59]

88
[✓ 38]

[✓ 43]
73

[✓ 23]
63

324

Which is less?

| | | | | |
|---|---|---|---|---|
| 36 | 46 | [✓ 26] | 76 | [✓ 16] |
| 49 | 89 | 59 | [✓ 39] | [✓ 29] |
| 63 | [✓ 23] | 73 | [✓ 53] | 93 |
| 72 | 82 | [✓ 52] | [✓ 22] | [✓ 42] |
| 95 | [✓ 85] | [✓ 35] | [✓ 65] | [✓ 25] |
| 66 | 76 | [✓ 36] | 96 | [✓ 26] |

Place a set of 32 (3 bundles of ten and 2 ones) and a set of 37 (3 bundles of ten and 7 ones) on a table or desk. Ask the children which set has more members. This may be determined by counting (tens and ones) and by pairing. Point to the set of 32.

How many tens? (3.)
How many ones? (2.)
What is the name of the number? (Thirty-two.)

Write 32 on the chalkboard and repeat the procedure with the set of 3 tens and 7 ones and write 37 under 32 on the chalkboard.

Which is greater, 32 or 37? (37.)
How do you know this is correct? (A set of 3 tens and 7 ones has more members than a set of 3 tens and 2 ones. 37 is greater than 32.)
Do these two numerals name the numbers that have the same number of tens? (Yes.)
Do they have the same number of ones? (No, a set of 37 has more ones than a set of 32.)

Leave these numerals on the chalkboard and repeat the procedure using 63 and 68, 92 and 98, etc. Record the pairs of numerals as you consider them.

Look at the numerals on the chalkboard.
The first pair is 32 and 37.
Which numeral names the greater number? (37.)
Can we tell just by looking at these two numerals that this is so? (Yes. The ten's digits are the same. Look at the one's digit. 7 is greater than 2 so 37 is greater than 32.)

Consider the other pairs of numerals in the same way. Help children generalize that if there are the same number of tens in the numbers then we need only compare the number of ones.

USING THE PUPIL'S BOOK, page 167-168:

Use the same procedure as you used for pages 163-164.

Repeat the same procedure and ask for the number which is less than a given one.

USING THE PUPIL'S BOOK, page 169-170:

Use the same procedure as you used for pages 163-164.

Which is greater?

21
[✓ 24]

34
[✓ 36]

[✓ 59]
53

[✓ 47]
45

62
[✓ 65]

[✓ 77]
74

[✓ 29]
26

83
[✓ 89]

35
[✓ 38]

91
[✓ 95]

[✓ 38]
36

[✓ 68]
64

Which is greater?

| | | | | |
|---|---|---|---|---|
| 23 | [✓ 27] | [✓ 25] | 20 | 22 |
| 45 | 42 | [✓ 47] | 44 | [✓ 49] |
| 76 | 71 | [✓ 77] | 75 | 74 |
| 32 | [✓ 39] | [✓ 36] | [✓ 33] | 31 |
| 87 | 82 | [✓ 88] | 83 | [✓ 89] |
| 54 | [✓ 59] | 53 | [✓ 57] | 51 |

Which is less?

| | | |
|---|---|---|
| [✓ 61]<br>65 | 45<br>[✓ 42] | [✓ 82]<br>86 |
| 98<br>[✓ 96] | 34<br>[✓ 33] | [✓ 57]<br>59 |
| [✓ 24]<br>27 | [✓ 67]<br>69 | 46<br>[✓ 43] |
| [✓ 75]<br>79 | 37<br>[✓ 32] | [✓ 81]<br>84 |

Which is less?

| | | | | |
|---|---|---|---|---|
| 33 | 38 | [✓ 32] | 35 | [✓ 31] |
| 65 | [✓ 62] | 68 | [✓ 64] | 66 |
| 88 | [✓ 81] | 89 | [✓ 83] | [✓ 87] |
| 57 | 58 | [✓ 52] | [✓ 56] | [✓ 53] |
| 23 | 29 | 25 | [✓ 21] | 24 |
| 49 | [✓ 43] | [✓ 45] | [✓ 47] | [✓ 42] |

- Write the numerals 16 and 6 on the chalkboard and ask the children to tell which names the greater number. Ask the children how they know that 16 is greater than 6. Try to obtain several different explanations. Use set materials to show that a set of 16 has more members than a set of 6. Represent 16 as 1 ten 6 ones and 6 as 6 ones. Place a number line (0-100) across the chalkboard so that the children can touch the numerals. Indicate that 16 is to the right of 6 on the number line. Then continue by asking whether all numbers in the teens are greater than the numbers which contain 0 tens. Ask whether every number in the fifties is greater than numbers in the thirties, and so on. Continue with specific examples:

> Which is greater, 25 or 17? (25.)
> Is two tens and five ones greater than one ten and 7 ones? (Yes.)
> Which is greater, 37 or 40? And so on.

In each case, restate the problem in terms of tens and ones and then use the number line to indicate the relative position of the numerals that name the numbers.

> If we have numbers between 10 and 99, which digits of the numerals should we look at first to help us decide which is the greater number? (The ten's digit.)
>
> If the numerals have the same ten's digit, how do we decide which number is greater? (The one's digit.)

Write numerals on the chalkboard and have children draw a ring around the numeral for the greater number. Have them tell you, if they hesitate, which digit of the numeral names the tens and the ones.

USING THE PUPIL'S BOOK, page 171:

Read the title. Direct the children to look at the first exercise. Ask which is greater 36 or 27? (36.) When the children give the correct answer direct them to draw a ring around 36. Children work independently on the rest of the exercises.

Page 172:

Use the same procedure for less than as you did for greater than on page 171.

Page 173:

Read the title. Direct the children's attention to the first exercise. Ask after it is read, What number is 1 greater than 23? (24).

When the children have given the correct answer, direct them to write 24 in the blank space. Children work independently on the rest of the exercises.

Page 174:

Use the same procedure as you did for page 173.

Which is greater?

[✓ 36] 27

79 [✓ 81]

[✓ 21] 19

[✓ 10] 6

53 [✓ 97]

17 [✓ 24]

77 [✓ 96]

[✓ 91] 58

45 [✓ 50]

[✓ 61] 59

[✓ 30] 13

[✓ 65] 32

[✓ 28] 23

88 [✓ 95]

[✓ 56] 49

Which is less?

| | | | | | |
|---|---|---|---|---|---|
| 21 | [✓ 19] | 95 | [✓ 88] | [✓ 27] | 36 |
| 91 | [✓ 58] | 97 | [✓ 53] | [✓ 79] | 81 |
| 28 | [✓ 23] | 61 | [✓ 59] | 17 | [✓ 13] |
| 10 | [✓ 6] | 56 | [✓ 49] | 30 | [✓ 13] |
| [✓ 45] | 50 | [✓ 32] | 61 | 65 | [✓ 32] |

Name the number.

1 greater than

23 is 24

45 is 46

32 is 33

54 is 55

76 is 77

98 is 99

67 is 68

81 is 82

1 less than

42 is 41

25 is 24

87 is 86

53 is 52

64 is 63

36 is 35

92 is 91

78 is 77

FURTHER ACTIVITIES:

1. Write a numeral on the chalkboard and have children name the number that is one greater or one less than the one whose name you have written. Ask them to name the number that is ten greater or less.

2. Let children use hundreds-square paper (10 rows of 10 squares) and write numerals from 0 through 99.

3. Numerals may be written either vertically or horizontally. To check understanding of greater than and less than, write on the chalkboard:

12, 21
45, 49
62, 57
38, 25
70, 39
98, 89

Ask children to copy the pairs of numerals and draw a ring around the one in each pair that names the greater number. (They will ring 21, 49, 62, 38, etc.) The activity may be varied by asking them to draw a ring around the numeral for the lesser number in each pair. On the other days you may wish to write 3 numerals in each group and ask children to draw a red ring around the numeral for the greatest number and a blue ring around the numeral for the least number.

4. Give each child three numeral cards and a piece of lined writing paper. The child is to arrange the three numeral cards in order from least to greatest number.

23 43 68

The child then copies the three numerals on his paper. When the first set is finished the child gets a new set of cards and repeats the activity for another set of cards. It is possible to modify this work by giving some children only two cards and other children as many as five cards.

Pages 175-176:

These are practice pages similar to those before. Use the same procedure as for page 163.

Page 177

Read the title. Direct the children to look at the first exercise. Ask

Which number is the least? (19.)
Where shall I put 19? (In the first box on the left.)
Which of 25 and 31 is less? (25.)
Where shall I put 25? (In the middle box.)
Which number is the greatest? (31.)
Where shall I put 31? (In the last box; in the box at the right.)

Pupils work independently on the other exercises.

Which is greater?

15
[✓ 51]

[✓ 61]
16

13
[✓ 31]

[✓ 21]
12

36
[✓ 63]

[✓ 71]
17

[✓ 65]
56

[✓ 81]
18

39
[✓ 93]

14
[✓ 41]

[✓ 76]
67

19
[✓ 91]

Which is less?

| | | |
|---|---|---|
| [✓ 17]<br>71 | 91<br>[✓ 19] | [✓ 12]<br>21 |
| 54<br>[✓ 45] | [✓ 15]<br>51 | [✓ 36]<br>63 |
| 31<br>[✓ 13] | 61<br>[✓ 16] | [✓ 14]<br>41 |
| [✓ 69]<br>96 | 71<br>[✓ 17] | [✓ 18]<br>81 |

340

## Order of Numbers

Write in Order from the Least to the Greatest

| | | | | | |
|---|---|---|---|---|---|
| 19 | 31 | 25 | 94 | 28 | 54 |
| 19 | 25 | 31 | 28 | 54 | 94 |
| 86 | 45 | 29 | 23 | 12 | 18 |
| 29 | 45 | 86 | 12 | 18 | 23 |
| 56 | 84 | 99 | 53 | 36 | 16 |
| 56 | 84 | 99 | 16 | 36 | 53 |
| 63 | 48 | 72 | 15 | 34 | 83 |
| 48 | 63 | 72 | 15 | 34 | 83 |

5. Game for three children. Each child starts with 20 numeral cards, any of the set 0 through 99; no duplicates. The cards are in a stack, face down. Each child turns one card face up. The children compare the three numbers named, and the child whose numeral card names the greatest number takes all three cards and puts them on the bottom of his stack.

## Chapter 7

## ADDITION AND SUBTRACTION

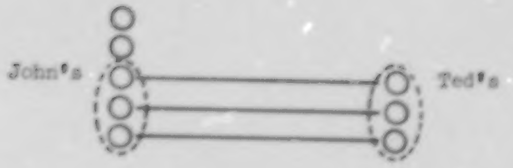
### BACKGROUND

You may want to review the background for Chapter 4, where the fundamental definitions concerning addition and subtraction were made. We recall, for example, that $3 + 5$ is the number of members in the set obtained by joining a set of $5$ members to a set of $3$ members and $5 - 3$ is the number of members remaining if a subset consisting of $3$ members is removed from a set of $5$.

The idea of partitioning sets is used in this chapter primarily for reinforcement of various number relationships, but we shall later use partitioning into equivalent sets in the discussion of place-value and division. Partitioning a set is just separating it into two disjoint subsets. For example, we may partition the set consisting of Mildred, Jean, Stan, and Mary into the set consisting of Mildred, Jean, and Stan, and the set consisting of Mary. (We shall later partition a set into more than two subsets.)

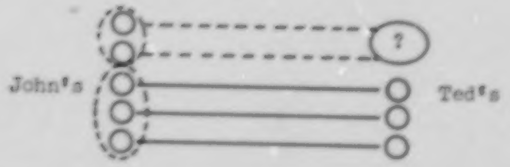
Partitioning is related to both joining and removing. For example, if we join the set consisting of Mildred, Jean, and Stan to the set consisting of Mary, we have the original set consisting of Mildred, Jean, Stan, and Mary. Because of the relation between joining and addition, we see that, in general, the number of members in the original set is equal to the sum of the number of members in the two sets of the partition (in this case, $4 = 1 + 3$ or $4 = 3 + 1$). Then, $4 - 3 = 1$, and in general, the number of members of the original set minus the number of members of one of the sets of the partition is the number of members of the other set of the partition.

There are also other problems which lead to subtraction equations. For example: If John has $5$ marbles and Ted has $3$ marbles, how many more marbles has John? We may think of pairing Ted's marbles with John's, as shown on the next page,



removing from the set of John's marbles a set which is equivalent to Ted's, and identifying the number of the remaining set. Since the number of Ted's marbles is equal to the number of the equivalent subset of John's marbles, we see that John has 5 - 3 more marbles than Ted.

The following is a closely related problem: if John has 5 marbles and Ted has 3 marbles, how many marbles must we give Ted so that he has as many as John? Schematically, we can pose the question as follows:

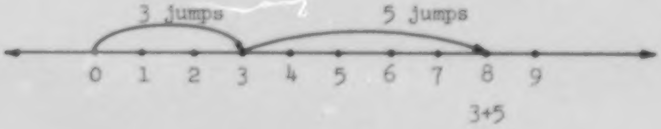


If we remove from the set of John's marbles a subset which is equivalent to Ted's set, then the remaining set is equivalent to the unknown set. We conclude that we must give Ted 5 - 3 marbles.

The last description of subtraction in terms of sets leads to a formulation which does not depend on sets, but only on the ideas of addition which we have already introduced. (Of course, the definition of addition does depend on manipulation of sets.)

Suppose again that John has 5 marbles and Ted has 3 marbles, and we wish to know how many marbles to give Ted so the boys will have the same number of marbles. The union of the set we give Ted and the set that Ted has must be equivalent to the set of John's marbles. Hence, the number of marbles we must give, which is 5 - 3, is the answer to the following question: 3 + ? = 5. In the same way, 4 - 2 is the answer to the question, 2 + ? = 4, and so on. This is sometimes called the missing addend description of subtraction. It is important that children work with this description as well as with the descriptions in terms of set manipulation since this will be the fundamental notion underlying the subtraction of numbers in the later grades. In general, we try to give the children experience with several of the ways that subtraction problems arise.

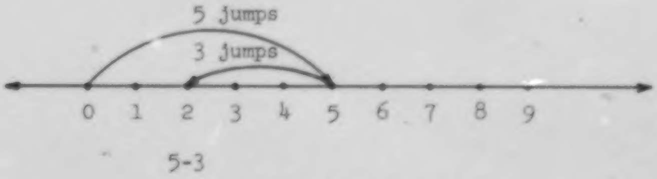
The number line is useful in learning about the operations of addition and subtraction. If we think of 0 as the starting point, then each numeral indicates the number of "jumps" required to get from the starting point to the point marked by the numeral. We may find the sum of 3 and 5 by taking 3 jumps, and then 5 jumps, and then reading the numeral (which indicates the number of jumps taken from the starting point).



$3 + 5 = 8$

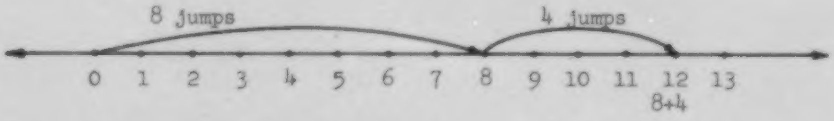
Notice that we do not have to count out the 3 jumps: the numeral "3" shows where counting 3 jumps would have gotten us.

The number line can also be used for subtraction: $5 - 3$ is the number of jumps from the starting point which results from taking 5 jumps forward and then 3 jumps backward.

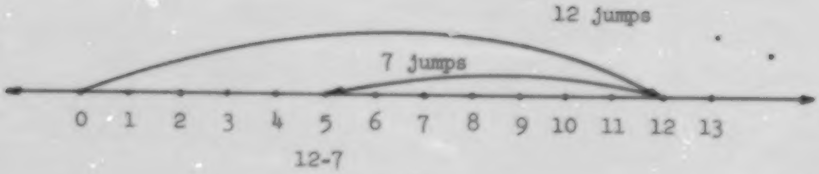


$5 - 3 = 2$

Finally, we note that the number line, which we use here primarily for reinforcement and variety, can be used in a very important way in introducing such problems as $8 + \square = 12$ and $12 - \square = 5$.



$8 + 4 = 12$


12 jumps
7 jumps
0
1
2
3
4
5
6
7
8
9
10
11
12
13
12-7

12 - 7 = 5

## 7-1. PARTITIONS AND ADDITION

**OBJECTIVE:** To reinforce and extend the child's understanding of addition by using partitions of sets.

**VOCABULARY:** Partition.

**MATERIALS:** Flannel board objects, yarn, set materials for the children.

**BACKGROUND NOTE:**

Partitioning a set into two subsets simply means separating it into two parts. Each member of the set with which you started then belongs to just one of the two subsets. The union of the two subsets is the set with which you started, and because of the relation between adding and joining, each partition gives us information on addition. Thus the fact that a set of 5 can be partitioned into a set of 2 and a set of 3 shows us that $5 = 2 + 3$ and $5 = 3 + 2$ (since joining and adding are both commutative.)

### SUGGESTED PROCEDURE

**PRE-BOOK ACTIVITIES:**

Place four cut-outs on the flannel board. Ask the children how many members are in the set. Use yarn or any other suitable item to separate this set into two subsets of 2 members each.

> I have used this yarn to separate or PARTITION this set into two subsets.
> How many members are in each subset? (2.)
> Could I partition this set into two other subsets? (Yes.)
> What could I do to show a different partition? (Move the yarn.)

Follow the suggestions of the children until all possible partitions of two subsets have been shown. You probably will have to suggest the partitions of 0, 4 and 4, 0.

Replace the set of 4 on the flannel board with a set of 5.

> What is the number of members of the set? (5.)

Place a piece of yarn across the flannel board to show a partition of 3 and 2 members.

> What is the number of members in one subset? (3.)
> What is the number of members in the other subset? (2.)
> If we join these two sets, how many members will be in the new set? (5.)

> We can add the number of members of the subsets to find the number of members in the set.
> What is the equation? (3 + 2 = 5.)

Write this equation on the chalkboard or use the flannel numerals and symbols to display this equation on the flannel board.

> We found the number of members in the set by adding 2 to 3.
> We wrote 3 + 2 = 5. Can we find the number of members in the set by adding 3 to 2? Can we write 2 + 3 = 5? (Yes.)

Record this equation and ask the children how the equations differ. The discussion should emphasize that the order in which the numbers are added does not affect the result. Therefore, two addition equations can be written for this partition.

Consider other partitions of a set of 5 members and write the addition equations associated with these partitions on the chalkboard as the children give them.

Continue the same procedure with sets of 6 through 9 members. For the partitions of each set, it is helpful to have the associated equations recorded on the chalkboard, e.g. 3 + 2 = 5, 2 + 3 = 5. This enables the children to see that a given set may be partitioned in several ways. In addition, children have the opportunity to develop the idea that a number may have several names.

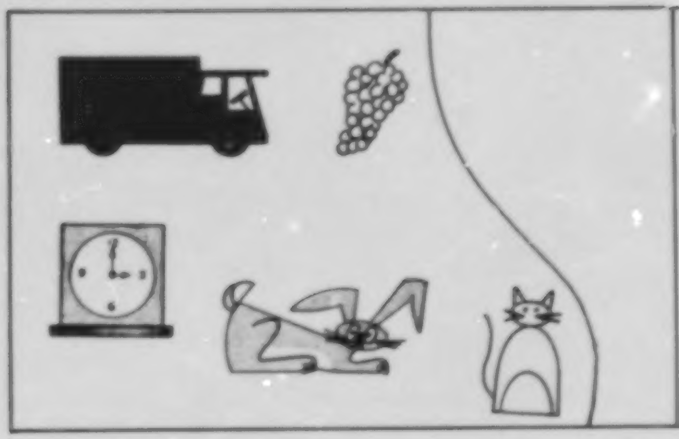
USING THE PUPIL'S BOOK, pages 178-182:

These pages show partitions of various sets. Ask the children to write equations for each partition. Note that if the set partition results in two equivalent subsets, only one addition equation can be written.

Give the children careful directions for using these pages. For example, on 178 direct their attention to the first set. Ask

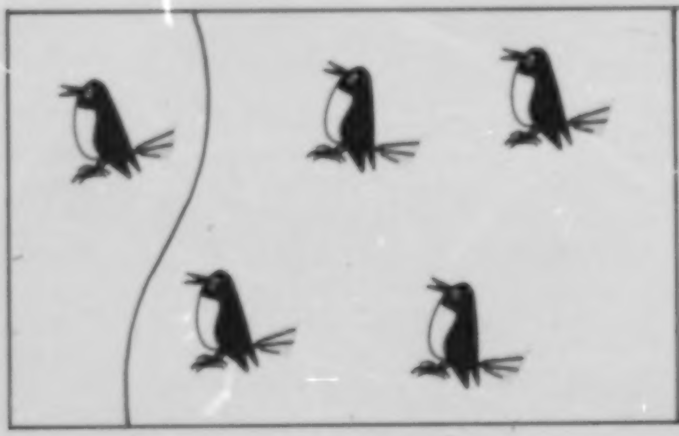
> What is the number in the set? (5.)
> How has the set been partitioned? (5 members and 0 members.)
> Who can write an equation for this partition of a set with 5 members?
> (5 + 0 = 5, 0 + 5 = 5.)

## Partitions



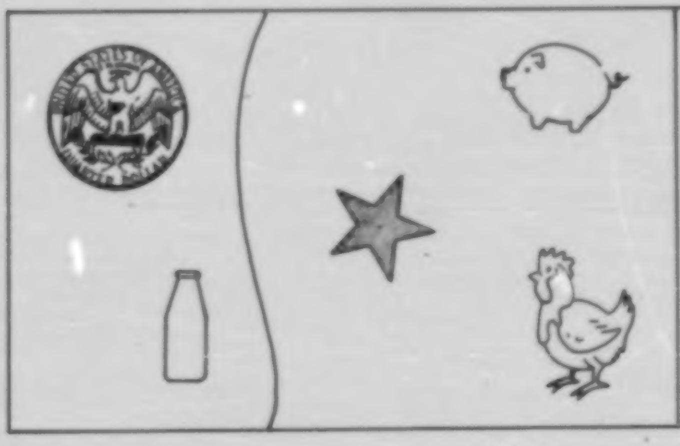
5 + 0 = 5

0 + 5 = 5



1 + 4 = 5

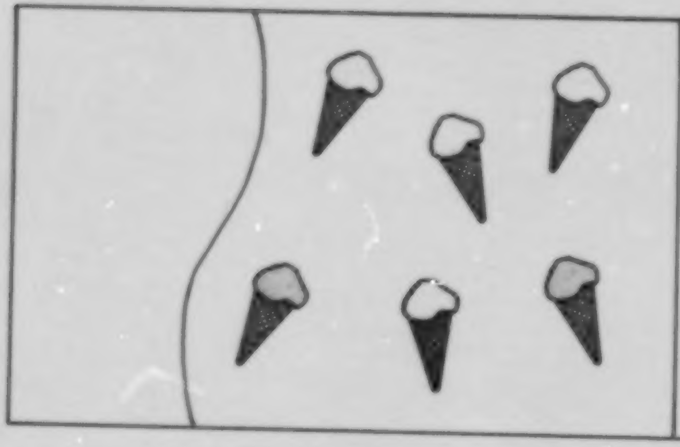
4 + 1 = 5



2 + 3 = 5

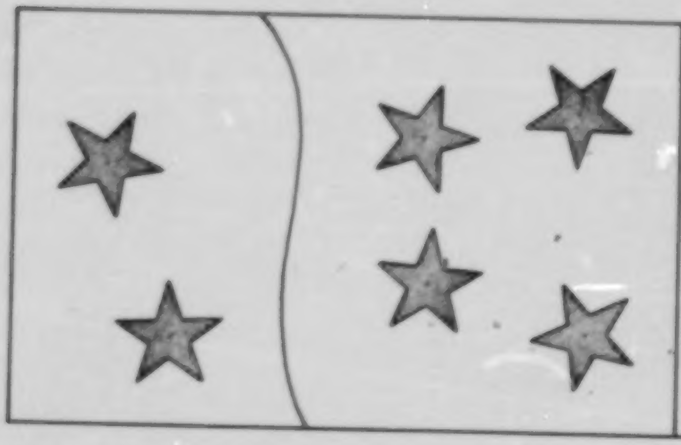
3 + 2 = 5

## Partitions



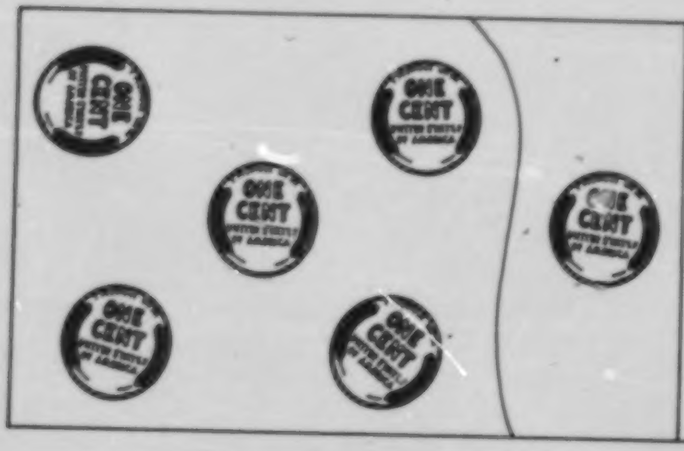
0 + 6 = 6

6 + 0 = 6



2 + 4 = 6

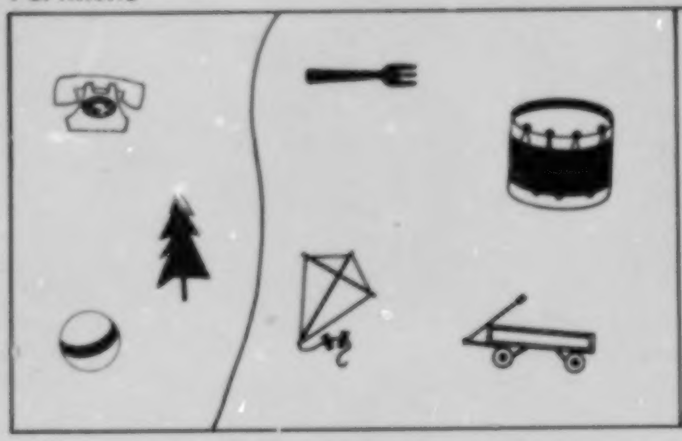
4 + 2 = 6


ONE CENT UNITED STATES OF AMERICA

5 + 1 = 6

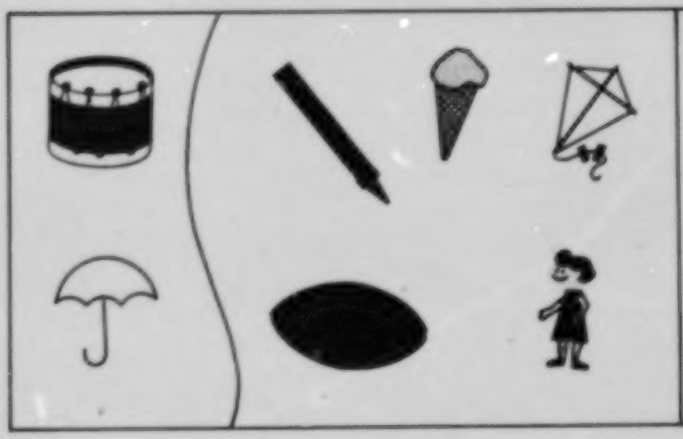
1 + 5 = 6

**Partitions**



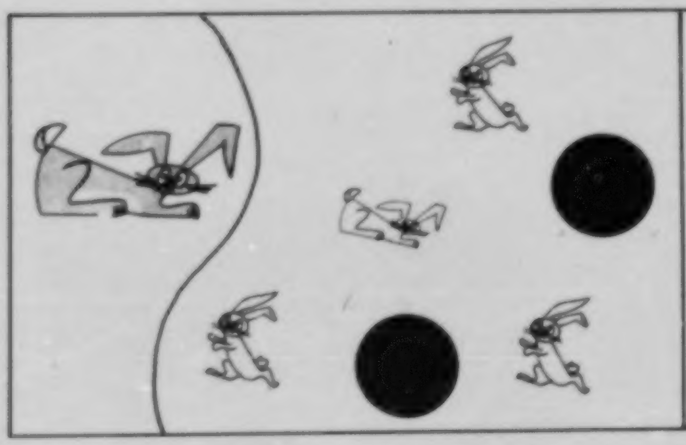
3 + 4 = 7

4 + 3 = 7



2 + 5 = 7

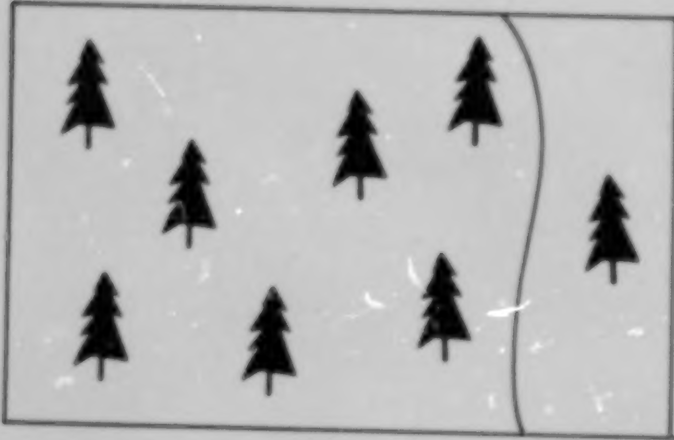
5 + 2 = 7



1 + 6 = 7

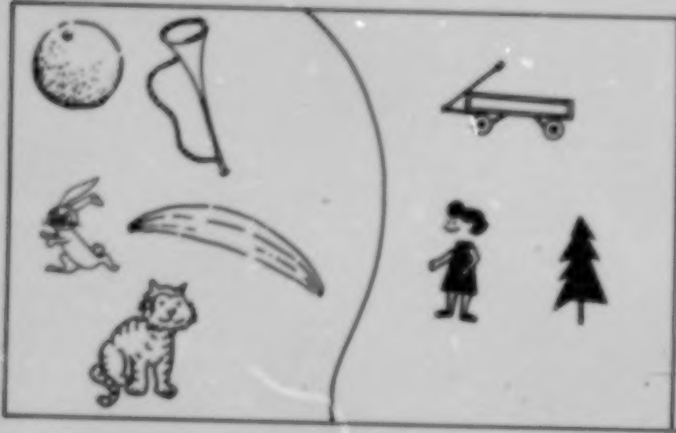
6 + 1 = 7

## Partitions



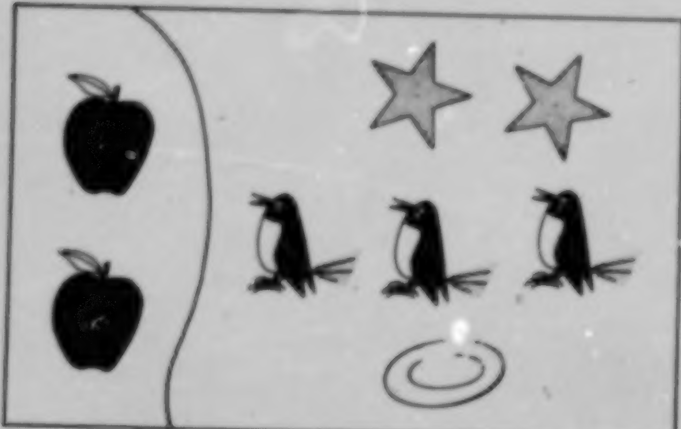
7 + 1 = 8

1 + 7 = 8



5 + 3 = 8

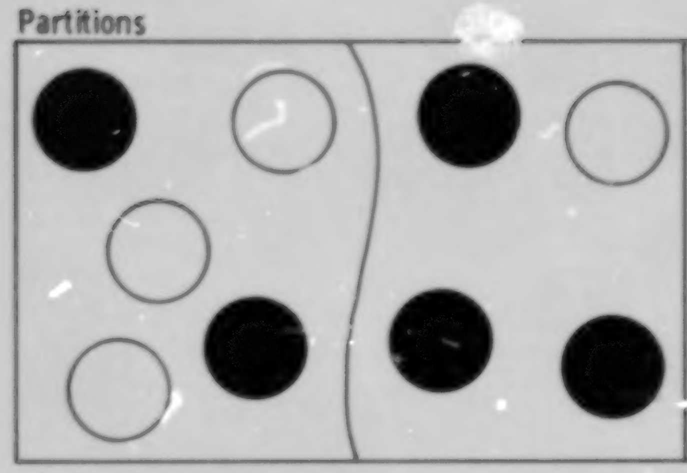
3 + 5 = 8



2 + 6 = 8

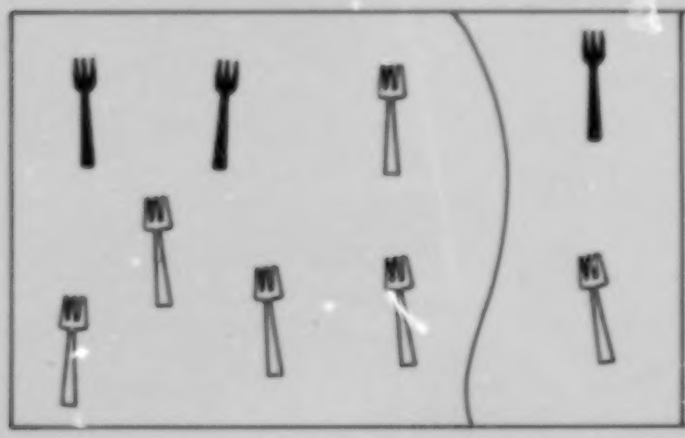
6 + 2 = 8

## Partitions



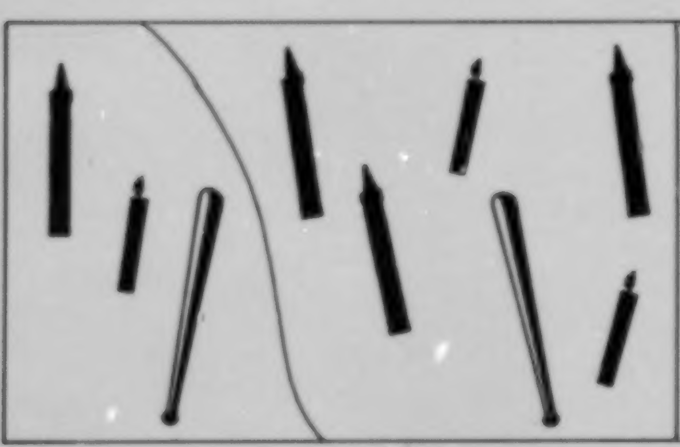
5 + 4 = 9

4 + 5 = 9



7 + 2 = 9

2 + 7 = 9



3 + 6 = 9

6 + 3 = 9

- Provide each child with a set of objects. Tell them you are thinking of a set of 4 that has been partitioned into 2 subsets and there are 3 members in one subset. Ask how many members are in the other subset. Demonstrate on the flannel board how children can use a set of 4 objects to answer your question.

Continue with similar questions concerning sets of 5 or 6 members. Have the children show with set objects how they would partition the set if given the number of members in one subset in order to determine the number of members in the other subset. When the technique is established, write equations such as:

$$5 = 3 + \square \qquad 6 = 2 + \square$$

$$5 = \square + 3 \qquad 6 = \square + 2$$

on the chalkboard. Have the children use the set objects to find the number of members in the other subset. Help the children see that each pair of equations can be completed from one partition.

USING THE PUPIL'S BOOK, pages 183-185:

Provide the children with set objects to aid them in completing the equation. Guide the children as they do the first equation. Then allow them to work independently.

COMPLETE THE EQUATIONS

$5 = 4 + \boxed{1}$

$5 = \boxed{1} + 4$

$5 = 2 + \boxed{3}$

$5 = \boxed{3} + 2$

$5 = 5 + \boxed{0}$

$5 = \boxed{0} + 5$

$6 = 6 + \boxed{0}$

$6 = \boxed{0} + 6$

$5 = 4 + \boxed{1}$

$6 = \boxed{4} + 2$

$6 = 1 + \boxed{5}$

$6 = \boxed{5} + 1$

COMPLETE THE EQUATIONS

$7 = 7 + \boxed{0}$

$7 = \boxed{0} + 7$

$7 = 5 + \boxed{2}$

$7 = \boxed{2} + 5$

$7 = 3 + \boxed{4}$

$7 = \boxed{4} + 3$

$7 = 6 + \boxed{1}$

$7 = \boxed{1} + 6$

$8 = 5 + \boxed{3}$

$8 = \boxed{3} + 5$

$8 = 8 + \boxed{0}$

$8 = \boxed{0} + 8$

$8 = 2 + \boxed{6}$

$8 = \boxed{6} + 2$

$8 = 7 + \boxed{1}$

$8 = \boxed{1} + 7$

COMPLETE THE EQUATIONS

$9 = 5 + \boxed{4}$

$9 = \boxed{4} + 5$

$9 = 8 + \boxed{1}$

$9 = \boxed{1} + 8$

$9 = 6 + \boxed{3}$

$9 = \boxed{3} + 6$

$9 = 2 + \boxed{7}$

$9 = \boxed{7} + 2$

$9 = 9 + \boxed{0}$

$9 = \boxed{0} + 9$

$10 = 7 + \boxed{3}$

$10 = \boxed{3} + 7$

$10 = 4 + \boxed{6}$

$10 = \boxed{6} + 4$

$10 = 10 + \boxed{0}$

$10 = \boxed{0} + 10$

$10 = 9 + \boxed{1}$

$10 = \boxed{1} + 9$

$10 = 8 + \boxed{2}$

$10 = \boxed{2} + 8$

## 7-2. PARTITIONS AND SUBTRACTION

**OBJECTIVE:** To reinforce and extend the child's understanding of subtraction and addition by using partitions of sets.

**VOCABULARY:** (No new terms.)

**MATERIALS:** Small objects.

**BACKGROUND NOTE:**

If a set of $5$ is partitioned into a set of $3$ and a set of $2$, and if the set of $2$ is removed, then the set of $3$ is the remaining set. We therefore see that $5 - 2 = 3$, because of the relation between subtraction and removing; by removing the set of $3$, in similar fashion, that $5 - 3 = 2$. Each partition thus leads to two subtraction equations. We have then $4$ related equations (two addition and two subtraction) for each partition into two non-equivalent subsets.

Throughout the following lesson, the children should use sets of small objects to aid them in finding sums and differences.

**SUGGESTED PROCEDURE:**

**PRE-BOOK ACTIVITIES:**

Give each child $6$ to $10$ objects. Ask him to partition the set on his desk into any two subsets. (There will be many different partitions.) Tell the children to pick up the objects in one of the subsets, thus removing it from the set.

*What set do you see now on the desk?* (The other subset.)

Ask each child to join the set still on his desk to the subset that he had removed.

*What set is on your desk?* (The set I started with.)

Ask children to make the same partition again. This time ask them to remove the other subset.

*What set is on your desk?* (The subset that we removed last time.)

Ask each to join the set he removed to the set that is on his desk.

*Do you have the set you started with?* (Yes.)

Discuss that removing either of the subsets of a partition leaves the other subset as the remaining set, and that joining the subset which had been removed to the remaining set results in the set with which they started.

- Give each child seven objects. Ask the children to partition the set of seven so that there are three members in one of the sets.

*How many are in the other set?* (4.)

*What equation can we write about this partition?* (7 = 3 + 4 and 7 = 4 + 3.)

*If you remove the set of 3, how many members are in the remaining set?* (4.)

*What equation suggests that you are removing a set of 3 from a set of 7?* (Hopefully, 7 - 3 = 4.)

Say that there are several equations that are suggested by this partition. Try to get the children to state them: 3 + 4 = 7, 4 + 3 = 7, 7 - 3 = 4, 7 - 4 = 3.

Continue working with partitions of 7 and identifying the 4 equations associated with each partition.

*USING THE PUPIL'S BOOK*, pages 186-189:

Direct attention to the first set. Ask,

*How many dots are in the set?* (5.)

*How many dots are in the set on the right?* (1.)

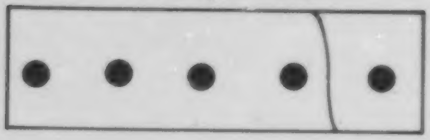
*How many dots are in the set on the left?* (4.)

*If we remove a set of 1 from a set of 5, how many are in the remaining set?* (4.)

*Who can write the equation for this partitioning?* (5 - 1 = 4, 5 - 4 = 1.)

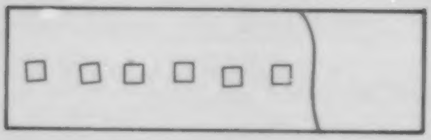
Children work independently on the rest of page.

Complete the equations.



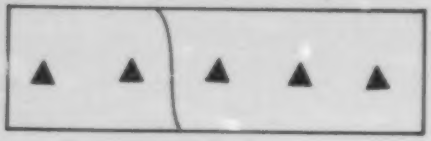
5 − 1 = 4

5 − 4 = 1



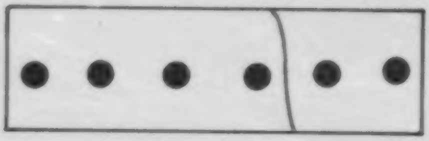
6 − 0 = 6

6 − 6 = 0



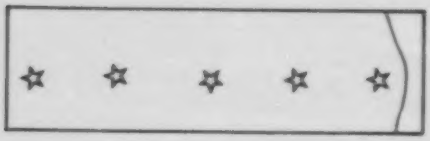
5 − 3 = 2

5 − 2 = 3



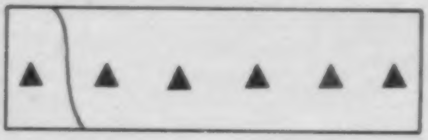
6 − 2 = 4

6 − 4 = 2



5 − 0 = 5

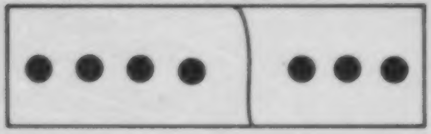
5 − 5 = 0



6 − 5 = 1

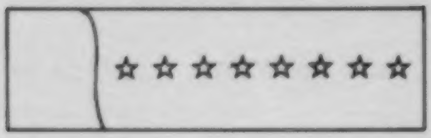
6 − 1 = 5

Complete the equations.



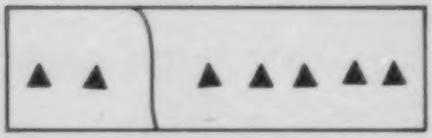
7 − 3 = 4

7 − 4 = 3



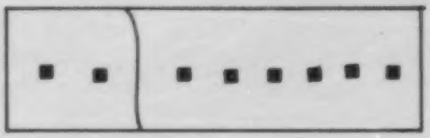
8 − 8 = 0

8 − 0 = 8



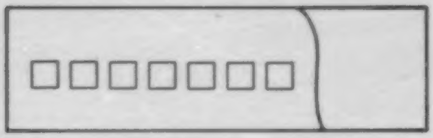
7 − 5 = 2

7 − 2 = 5



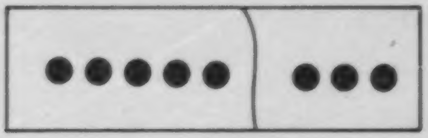
8 − 6 = 2

8 − 2 = 6



7 − 0 = 7

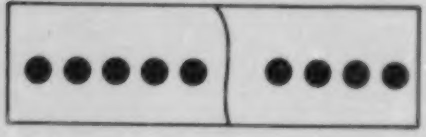
7 − 7 = 0



8 − 3 = 5

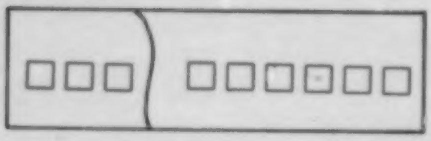
8 − 5 = 3

Complete the equations.



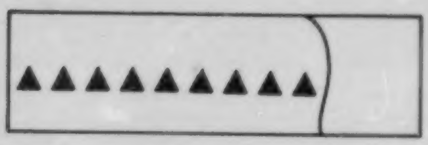
9 − 4 = 5

9 − 5 = 4



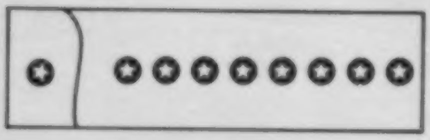
9 − 6 = 3

9 − 3 = 6



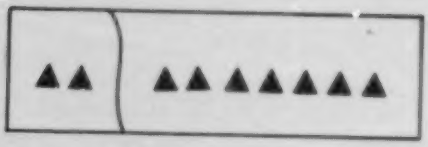
9 − 0 = 9

9 − 9 = 0



9 − 8 = 1

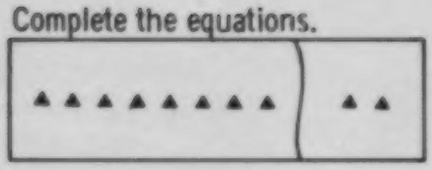
9 − 1 = 8



9 − 7 = 2

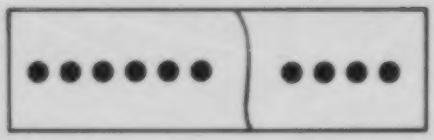
9 − 2 = 7

Complete the equations.



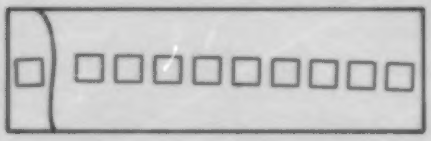
10 − 2 = 8

10 − 8 = 2



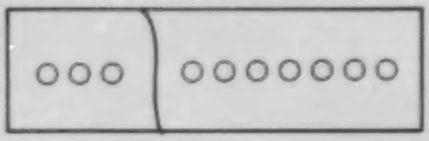
10 − 4 = 6

10 − 6 = 4



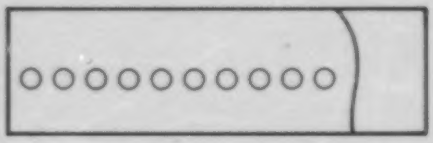
10 − 9 = 1

10 − 1 = 9



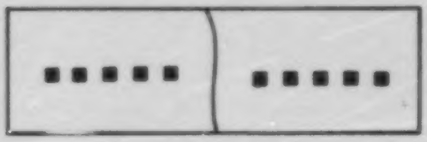
10 − 7 = 3

10 − 3 = 7



10 − 0 = 10

10 − 10 = 0



10 − 5 = 5

- Play "Acting Out Number Stories". Ask a group of six children to come to the front of the class to act out stories. For example: To act out 6 = 2 + 4, the children separate into a group of 2 and a group of 4, and then the groups come together. (2 + 4 = 6 or 4 + 2 = 6.) To act out 6 - 2 = 4, they begin in one set, and then a set of 2 children move away from the set of 6. To act out a partition of 6 into a set of 2 and a set of 4, they begin in a set of 6, then a set of 2 moves one way and a set of 4 moves the other way. (There are four equations for this play!) Ask the other children to hold up their hands as soon as they think of an equation that tells about the play, or later, ask them to write the equation on paper.

Dramatize problems such as the following. Children may use manipulative materials to represent the objects in each story problem.

Four girls and three boys were playing kickball.
How many children were playing kickball?
Can you make an equation about the story?

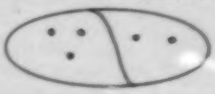
Billy had 8 marbles.
He shared his marbles with John.
Billy kept 5 marbles.
How many marbles did John get?
Can you make an equation about the marbles?

Sally was helping her mother set the table.
She carried 4 plates to the table.
Then she went back to the kitchen to get 3 more plates.
How many plates did Sally put on the table?
Can you make an equation about the plates?

Bill had 8 marbles.
He lost 5 of them.
How many marbles did he have left?
Can you make an equation about Bill's marbles?

Polly had 9 crayons.
She broke 4 of them.
How many good ones did she have left?
Can you make an equation about Polly's crayons?

- Draw the picture below on the chalkboard



__ + __ = __

__ + __ = __

What addition equation can we write about this partition? (3 + 2 = 5 or 2 + 3 = 5.)

Write the first equation given in the space provided.

We know there are 2 addition equations for this partition. Tell me how to write the second one. (Children might say, "Change the 2 and 3 around" or "Turn the numerals around.")

After writing the second equation, ask the children how many subtraction equations can be written for this partition. (2.) Provide spaces to write these equations on the chalkboard to the right of the addition equations.

How many members in the two subsets? (5.)
What numeral should I write in the first space in each of these equations? (5.)
If I remove the subset of 3, how many members in the remaining set? (2.)

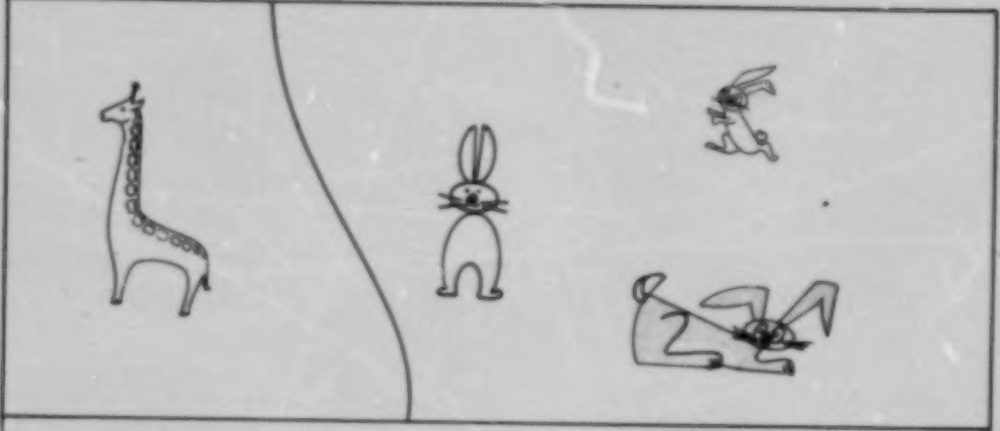
Complete this equation. Then ask the same question about the subset of 2 and complete the other equation.

Repeat the same procedure with several other partitions. Children accept the fact that only one addition and one subtraction equation can be written for any partition of 2 equivalent subsets without any difficulty. You may want to use such partitions at this time, or you may want to wait until you have established the fact that 2 addition and 2 subtraction equations can be written for a partition resulting in 2 nonequivalent subsets.

USING THE PUPIL'S BOOK, pages 190-195:

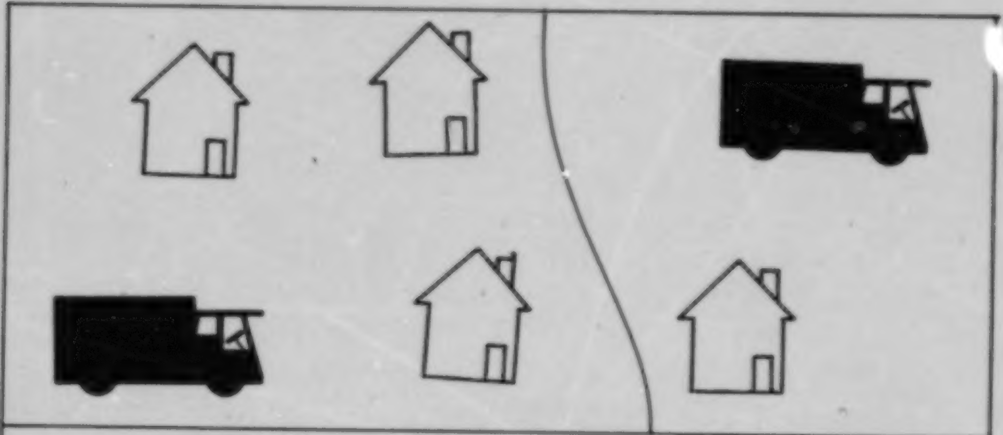
These pages show partitions of various sets. Ask the children to write equations for each partition.

## Equations



1 + 3 = 4     4 – 3 = 1

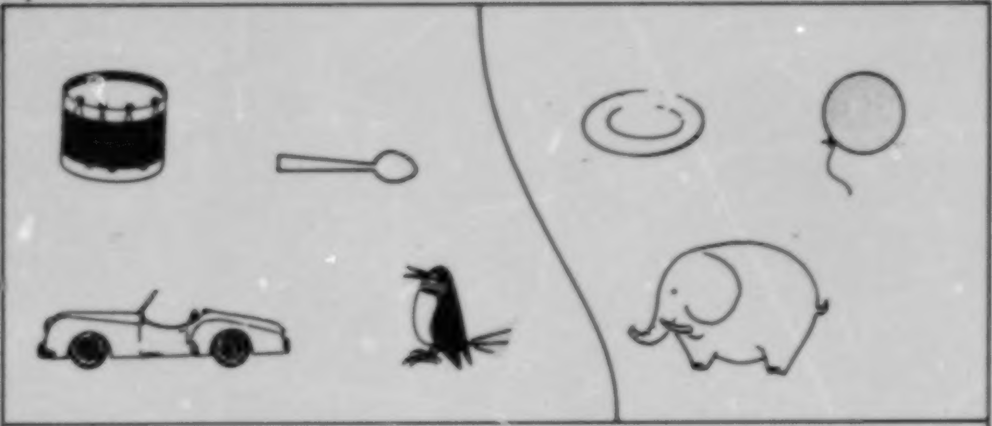
3 + 1 = 4     4 – 1 = 3



4 + 2 = 6     6 – 2 = 4

2 + 4 = 6     6 – 4 = 2

## Equations

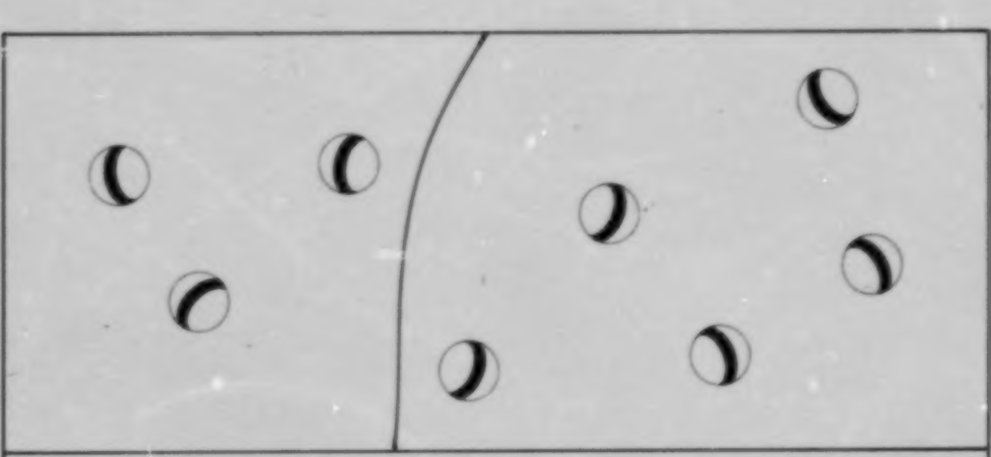


4 + 3 = 7

3 + 4 = 7

7 - 3 = 4

7 - 4 = 3



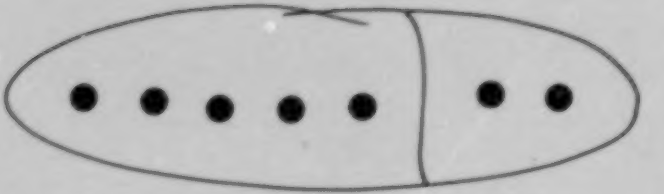
3 + 5 = 8

5 + 3 = 8

8 - 5 = 3

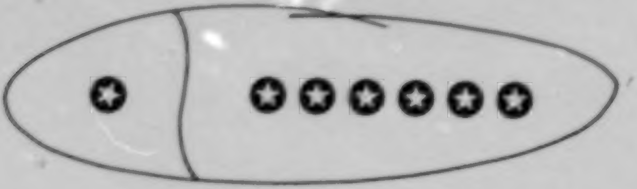
8 - 3 = 5

## Equations



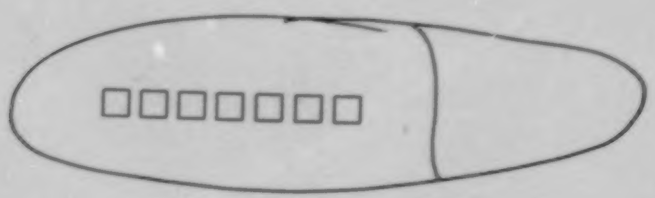
5 + 2 = 7  7 - 2 = 5

2 + 5 = 7  7 - 5 = 2



1 + 6 = 7  7 - 6 = 1

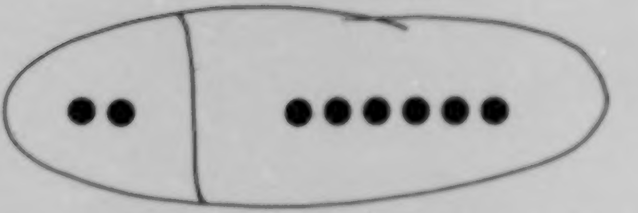
6 + 1 = 7  7 - 1 = 6



7 + 0 = 7  7 - 0 = 7

0 + 7 = 7  7 - 7 = 0

## Equations

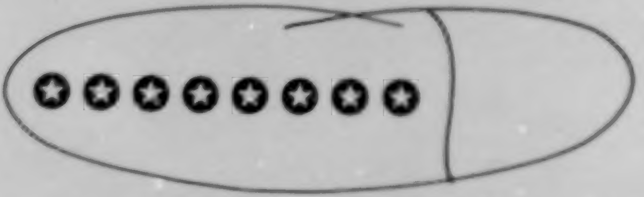


2 + 6 = 8

6 + 2 = 8

8 − 6 = 2

8 − 2 = 6

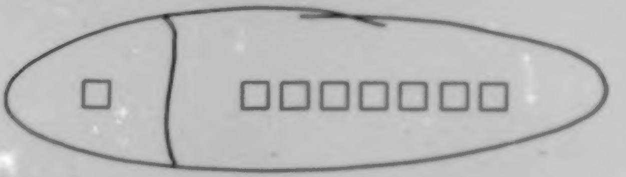


8 + 0 = 8

0 + 8 = 8

8 − 0 = 8

8 − 8 = 0



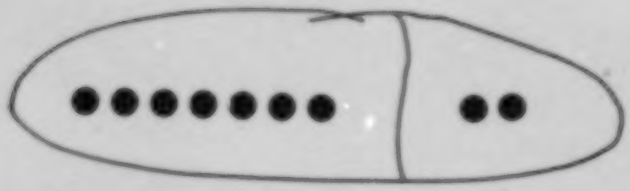
1 + 7 = 8

7 + 1 = 8

8 − 7 = 1

8 − 1 = 7

## Equations

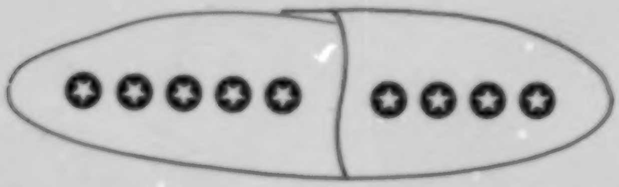


7 + 2 = 9

2 + 7 = 9

9 − 2 = 7

9 − 7 = 2

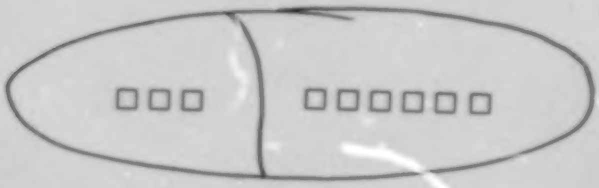


5 + 4 = 9

4 + 5 = 9

9 − 4 = 5

9 − 5 = 4



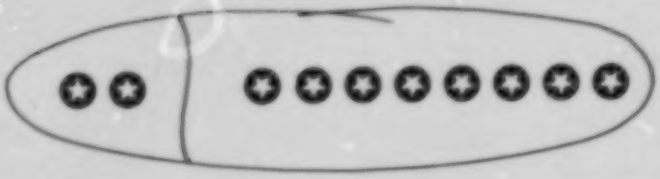
3 + 6 = 9

6 + 3 = 9

9 − 6 = 3

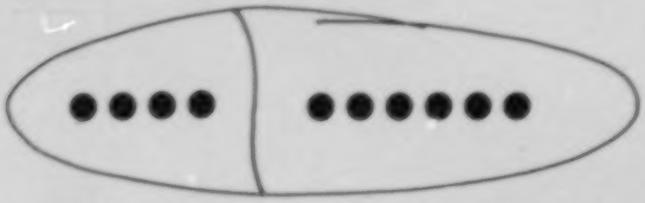
9 − 3 = 6

## Equations



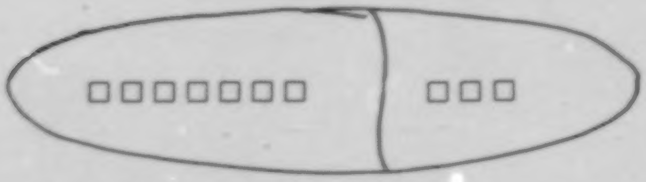
2 + 8 = 10
8 + 2 = 10

10 − 8 = 2
10 − 2 = 8



4 + 6 = 10
6 + 4 = 10

10 − 6 = 4
10 − 4 = 6



7 + 3 = 10
3 + 7 = 10

10 − 3 = 7
10 − 7 = 3

Pages 196-197:

Give each child a set of 9 objects. Read the title at the left. Tell the children that they are to draw a ring around the sum which names 7. Do the same for the title at right. Tell the children to use their objects to find the set when they need to.

Pages 198-200:

Give each child a set of 10 objects to use to find the differences. On these pages, the pupils are to ring the differences that name the number listed at the top of the page. Use the same procedure as on pages 196-197.

Which are equal to 7?

[✓ 3 + 4]

4 + 4

8 + 1

2 + 7

5 + 3

[✓ 6 + 1]

9 + 0

1 + 8

[✓ 2 + 5]

Which are equal to 8?

[✓ 7 + 1]

5 + 2

[✓ 6 + 2]

[✓ 3 + 5]

4 + 3

5 + 4

1 + 6

[✓ 4 + 4]

[✓ 8 + 0]

Which are equal to 9?

[✓ 8 + 1]

8 + 2

[✓ 5 + 4]

2 + 6

[✓ 3 + 6]

10 + 10

[✓ 1 + 8]

[✓ 7 + 2]

[✓ 9 + 0]

Which are equal to 10?

6 + 2

[✓ 5 + 5]

4 + 5

[✓ 2 + 8]

3 + 4

[✓ 3 + 7]

0 + 9

[✓ 4 + 6]

[✓ 1 + 9]

Which are equal to 5?

[✓ 6 − 1]

9 − 3

[✓ 8 − 3]

4 − 2

10 − 8

9 − 4

6 − 2

[✓ 5 − 0]

8 − 1

Which are equal to 6?

7 − 2

[✓ 10 − 4]

9 − 1

6 − 4

[✓ 8 − 2]

5 − 4

[✓ 7 − 1]

10 − 2

7 − 4

Which are equal to 7?

[✓ $8 - 1$]

$6 - 3$

$7 - 5$

[✓ $9 - 2$]

$4 - 4$

$9 - 3$

[✓ $10 - 3$]

$7 - 2$

[✓ $7 - 0$]

Which are equal to 8?

$8 - 3$

[✓ $10 - 2$]

$6 - 4$

$7 - 1$

[✓ $8 - 0$]

$10 - 4$

$5 - 3$

[✓ $9 - 1$]

$6 - 6$

| Which are equal to 9? | Which are equal to 10? |
|---|---|
| 7 − 4 | 9 − 2 |
| 8 − 6 | 10 − 10 |
| [✓ 10 − 1] | 7 − 5 |
| 6 − 5 | 8 − 1 |
| 8 − 7 | 9 − 5 |
| [✓ 9 − 0] | [✓ 10 − 0] |
| 10 − 5 | 8 − 6 |
| 9 − 9 | 10 − 4 |
| 8 − 4 | 9 − 4 |

__Page__ 201:

Direct the pupils to color the box after 7, red; after 8, blue; and after 9, yellow. Now give the directions: Ring all names for 7 in red, all names for 8 in blue and all names for 9 in yellow.

## Names for Numbers

| 7 red | 8 blue | 9 yellow |
|---|---|---|
| red 3 + 4 | blue 4 + 4 | blue 0 + 8 |
| yellow 0 + 9 | blue 10 − 2 | yellow 7 + 2 |
| yellow 5 + 4 | 3 + 3 | 8 + 2 |
| red 10 − 3 | yellow 8 + 1 | 3 + 1 |
| blue 2 + 6 | blue 5 + 3 | yellow 10 − 1 |
| red 8 − 1 | 7 + 3 | 5 + 1 |
| red 2 + 5 | red 6 + 1 | yellow 6 + 3 |
| red 9 − 2 | blue 9 − 1 | 10 − 0 |

## 7-3. ADDITION AND SUBTRACTION ON THE NUMBER LINE.

OBJECTIVE: To extend the children's understanding of addition and subtraction by using the number line.

VOCABULARY: (No new words.)

MATERIALS: (Possibly, construction paper for number line.)

### SUGGESTED PROCEDURE

### PRE-BOOK ACTIVITY:

### ADDITION:

The number line (which was introduced in Chapter 2, Section 3) may be used as an aid in addition and subtraction.

Review carefully the fact that the numeral on the line shows the number of jumps from the starting point, 0.

> Find the point on the number line which shows that you have taken 8 jumps from the starting point, 0.
> Find the point which shows that you have taken 3 jumps, from the starting point, 0.
> Find the point which shows that you have taken 3 jumps and then 2 jumps, from the starting point 0.

If this is difficult, ask a child to point to the place on the number line which shows that he has taken 3 jumps from the starting point. Use a pointer or pencil to indicate the point on the line. Ask him not to go back to the starting point but to take two more jumps. (It may be helpful to think of this as a "stop for a rest". A number line drawn on the floor can be used if necessary to dramatize this idea.)

> Now that you have taken 3 jumps and 2 jumps from the starting point, where are you on the number line? (5.)
> This tells us that three plus two equals five.

Write the equation, $3 + 2 = 5$.

> If we have a word problem, we can solve it on the number line if we know the numbers which we want to add.

> Jerry had 5 books
> He bought 2 books
> How many books does he have?
> What is the number of books which Jerry had to begin with? (5.)

Ask a child to take the same number of jumps from the starting point on a number line as the books Jerry had.

How many books did Jerry buy? (2.)

Ask the child to take that many more jumps on the number line, this time starting at 5.

What is the number of jumps that you have taken? (7.)
We wanted to add 5 and 2.
When we take those numbers of jumps on the number line we find that we have taken 7 jumps.
The set of 5 books and 2 books, if joined would be the same number of books. Thus, 5 + 2 = 7.

## SUBTRACTION

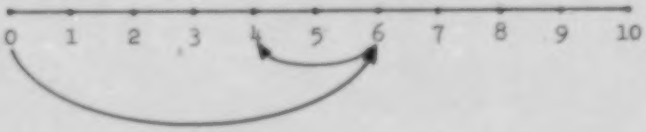
Draw a number line on the chalkboard. Write the eqatuion, 6 - 2 = ___.

To complete this equation we must first take six jumps on the number line.
Start at 0.

Ask a child to do this and either to mark the point where he stops or to draw the curve which shows the number of jumps.

To solve the equation, we must subtract 2 from 6.
On the number line, this means we will have to go back 2 jumps.

Ask a child to touch the point which shows where you would be if you went back 2 jumps from 6. Draw the curve which shows you have gone back 2 jumps.



Where did we stop this time? (4.)
When we went 6 steps forward and back 2 steps we stopped at 4.
This is one way of showing that 6 minus 2 equals 4.

It may be necessary to use a line which is drawn on the floor for the first work with subtraction on the number line. This would enable children to take jumps forward and then to come back on the number line.

Some children have difficulty using the number line to complete equations because they are confused by the numerals at which they are look while they count the second set of jumps.

Two ways to use the number line are given here. You may want to develop one to a great extent and exclude the other or try to use both of the ideas suggested.

Make a number line on a piece of cardboard or oaktag 12" x 36". This should be large enough for all children to see. Fold the top of the paper to cover the numerals.

Place a pencil on the starting point (Figure 1). (0.) Take jumps on the number line which correspond to the set to which another set is joined in the problem. (Some children will make one jump at a time (Figure 2) while other children will go to five in one jump (Figure 3).)

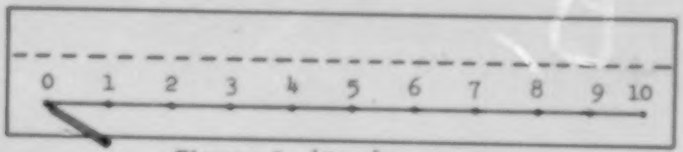


Figure 1 (Open)

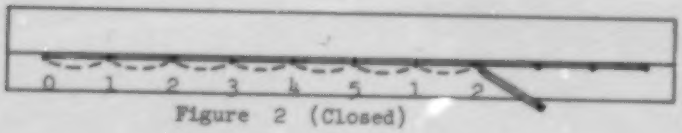


Figure 2 (Closed)

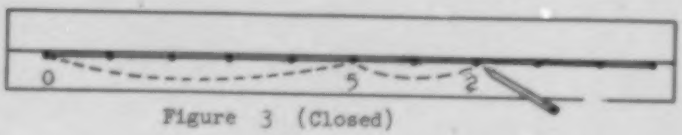


Figure 3 (Closed)

Move the pencil to that point. Some children may need to move the pencil as they take each jump. Keep the pencil on the last point and unfold the page. The pencil will be on the point which tells the total number of jumps. (Figure 4)

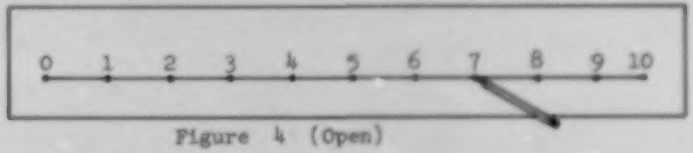


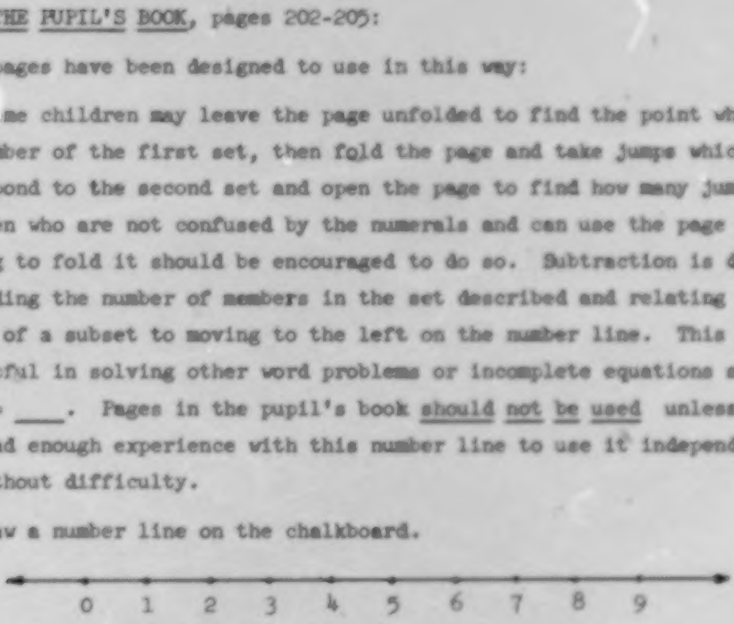
Figure 4 (Open)

USING THE PUPIL'S BOOK, pages 202-205:

These pages have been designed to use in this way:

Some children may leave the page unfolded to find the point which shows the number of the first set, then fold the page and take jumps which correspond to the second set and open the page to find how many jumps in all. Children who are not confused by the numerals and can use the page without needing to fold it should be encouraged to do so. Subtraction is developed by finding the number of members in the set described and relating the removing of a subset to moving to the left on the number line. This page may be helpful in solving other word problems or incomplete equations such as $3 + 2 = \_\_\_\_$. Pages in the pupil's book should not be used unless children have had enough experience with this number line to use it independently and without difficulty.

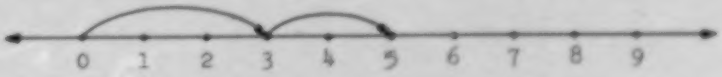
2. Draw a number line on the chalkboard.



Write an equation, $3 + 2 = \_\_\_$.

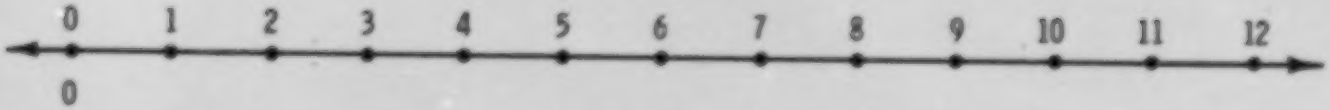
Place the chalk on the starting point. Ask a child to put a finger on the point which tells the number of jumps corresponding to the number of the first set. (The child should touch 3.) Draw a curve from the starting point to the point the child is touching.

Now, using 3 as the starting point ask where 2 more jumps would take us. (Child should now touch 5.) Draw a curve from the 3 to the 5. This represents the second set of jumps.



The point where the last curve ends tells the number of jumps and enables the children to complete the equation.

Fold until edge covers numerals, but not dots.


0
1
2
3
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0

Complete the equations.

$2 + 1 = \boxed{3}$

$5 + 1 = \boxed{6}$

$1 + 5 = \boxed{6}$

$1 + 2 = \boxed{3}$

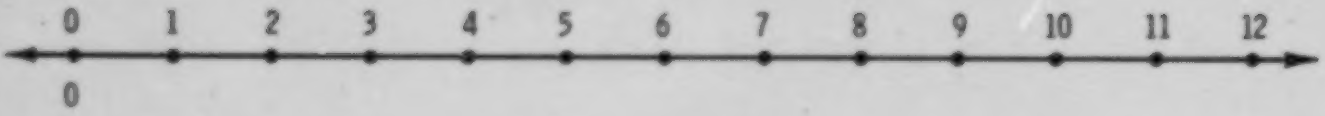
$5 + 3 = \boxed{8}$

$2 + 4 = \boxed{6}$

$1 + 6 = \boxed{7}$

$3 + 4 = \boxed{7}$

Fold until edge covers numerals, but not dots.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Complete the equations.

$3 + 2 = \boxed{5}$

$4 + 5 = \boxed{9}$

$4 + 2 = \boxed{6}$

$2 + 5 = \boxed{7}$

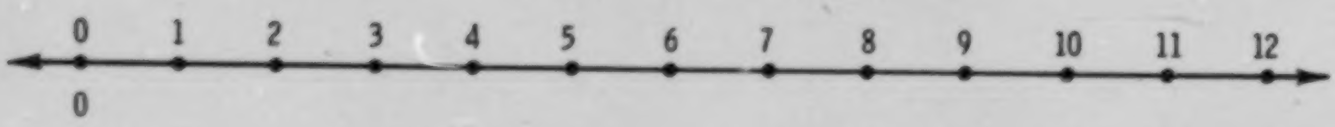
$1 + 7 = \boxed{8}$

$6 + 2 = \boxed{8}$

$3 + 5 = \boxed{8}$

$4 + 3 = \boxed{7}$

Fold until edge covers numerals, but not dots.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$5 - 3 = \boxed{2}$

$8 - 5 = \boxed{3}$

$7 - 2 = \boxed{4}$

$8 - 2 = \boxed{6}$

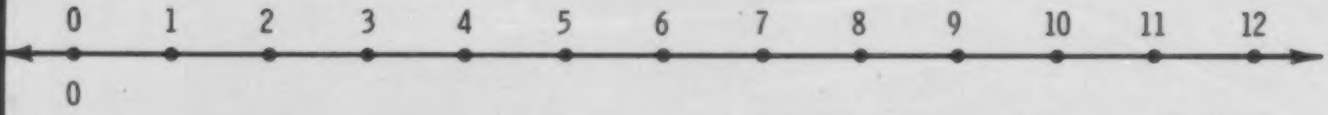
$9 - 7 = \boxed{2}$

$6 - 1 = \boxed{5}$

$5 - 2 = \boxed{3}$

$8 - 2 = \boxed{6}$

Fold until edge covers numerals, but not dots.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Complete the equations.

$8 - 1 = \boxed{7}$

$3 + 6 = \boxed{9}$

$10 - 3 = \boxed{7}$

$7 - 6 = \boxed{1}$

$2 + 7 = \boxed{9}$

$9 - 5 = \boxed{4}$

$1 + 4 = \boxed{5}$

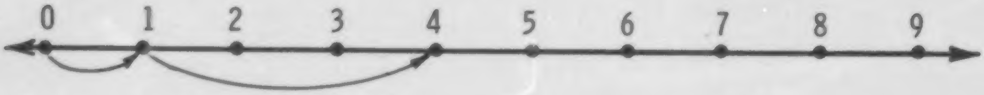
$2 + 6 = \boxed{8}$

Pages 206-208:

These pages are designed to be used in this way.

Heavy plastic taped over cardboard on which a number line has been drawn can be useful as an aid to independent work. The child can mark with a crayon the curve which shows the jumps he has taken and then remove the mark with a paper towel and use the same number line again.

Use the number line to help you complete the equation.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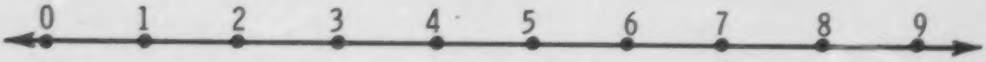
$$1 + 3 = \boxed{4}$$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$$2 + 1 = \boxed{3}$$

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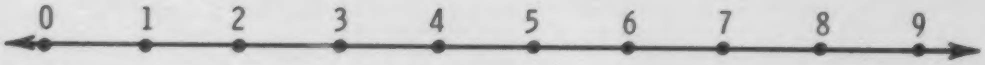
$$8 + 1 = \boxed{9}$$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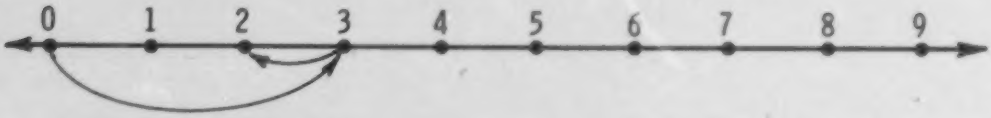
$$4 + 5 = \boxed{9}$$

Use the number line to help you complete the equation.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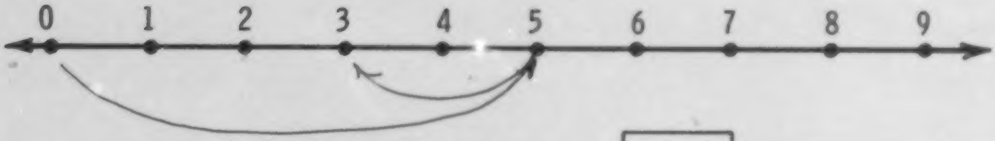
$$3 - 1 = \boxed{2}$$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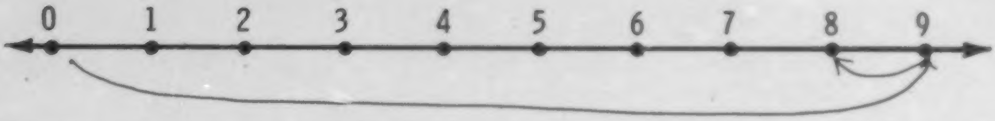
$$5 - 2 = \boxed{3}$$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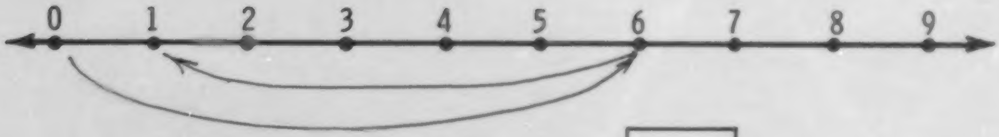
$$9 - 1 = \boxed{8}$$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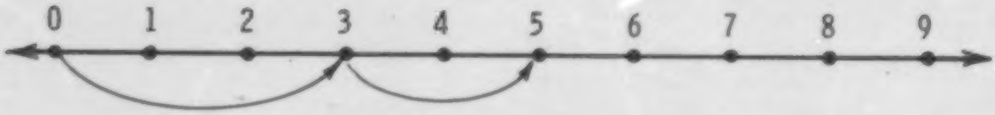
$$6 - 5 = \boxed{1}$$

Use the number line to help you complete the equation.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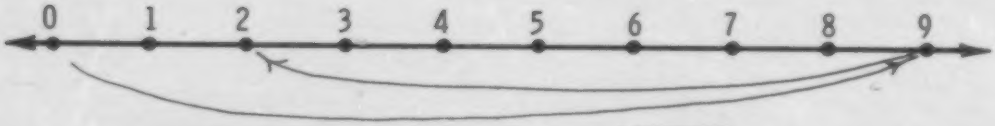
$$3 + 2 = \boxed{5}$$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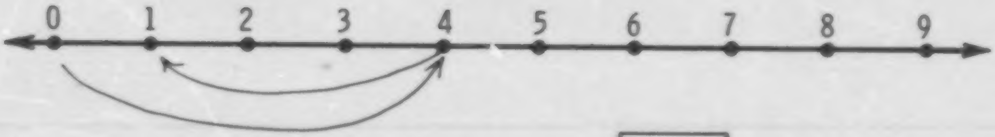
$$9 - 7 = \boxed{2}$$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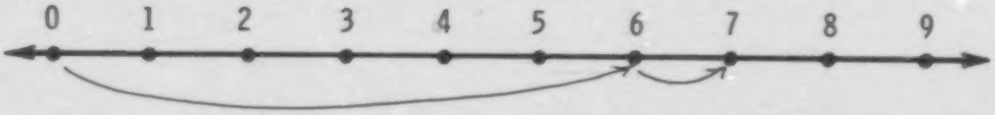
$$4 - 3 = \boxed{1}$$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$$6 + 1 = \boxed{7}$$

## 7-4. HOW MANY MORE?

**OBJECTIVE:** To find how many more members there are in one set than in another set.

**VOCABULARY:** (No new words.)

**MATERIALS:** Material, such as apples and lemons for flannel board display, flannel pairing symbols, two sets of numeral cards (sandpaper or flannel strips on the back of the card will make it stay on the flannel board).

**BACKGROUND NOTE:**

Recall that when given 2 sets, A and B, A has more members than B if, when the members of A are paired with members of B, there are members of A left over. In this lesson, we continue development of this idea by using subtraction to find how many more members there are in the first set. We say there are 2 more members in a set of 5 than in a set of 3 because, if we pair members of the set of 5 with members of the set of 3, there are 2 members left over. We also say there are 2 fewer members in the set of 3.

### SUGGESTED PROCEDURE

**PRE-BOOK ACTIVITIES:**

Place a set of 3 apples and a set of 5 lemons on the flannel board. Arrange the members of these sets in such a way that it is obvious without doing the actual pairing that one set has more members than the other.

Which set has more members? (Set of lemons.)
How many more? (2.)
Could you answer this question without counting all the lemons? (Yes.)
How did you do this? (You can see this is so. If you paired, all the apples would be paired with lemons, but there would be 2 lemons left over.)

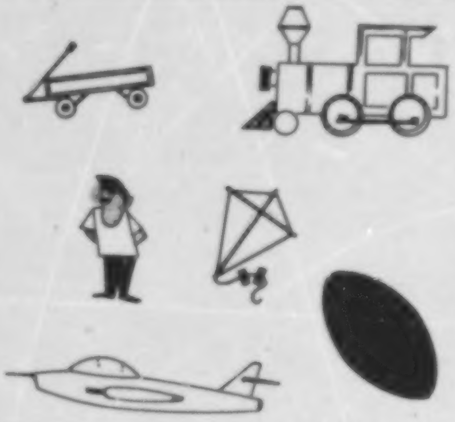
Agree and then ask a child to show the pairing with the pairing symbols or with yarn. Explain that we say that there are 2 more because, if we remove the subset of lemons which matches the set of apples, the remaining set has 2 members.

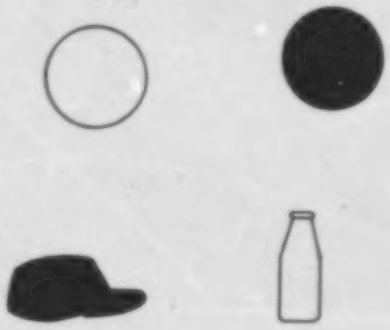
Continue the same procedure with sets of 7 and 9, 7 and 10, etc., until you are certain that the children understand the technique.

USING THE PUPIL'S BOOK, pages 209-212:

Read the questions on these pages to the children. Ask the children to first find out how many members are in each set and then record the numbers. Then ask them to pair the members of the set on the left with the members of the set on the right. They are then to decide which set has more members and how many more.

## Comparing Sets

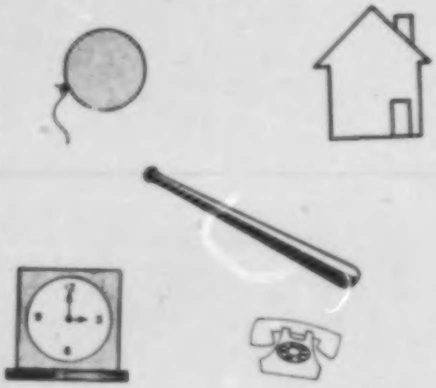


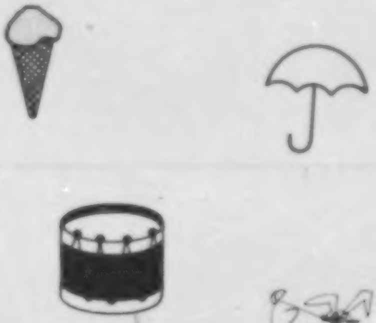


How many? 6

How many? 4

How many more? 2



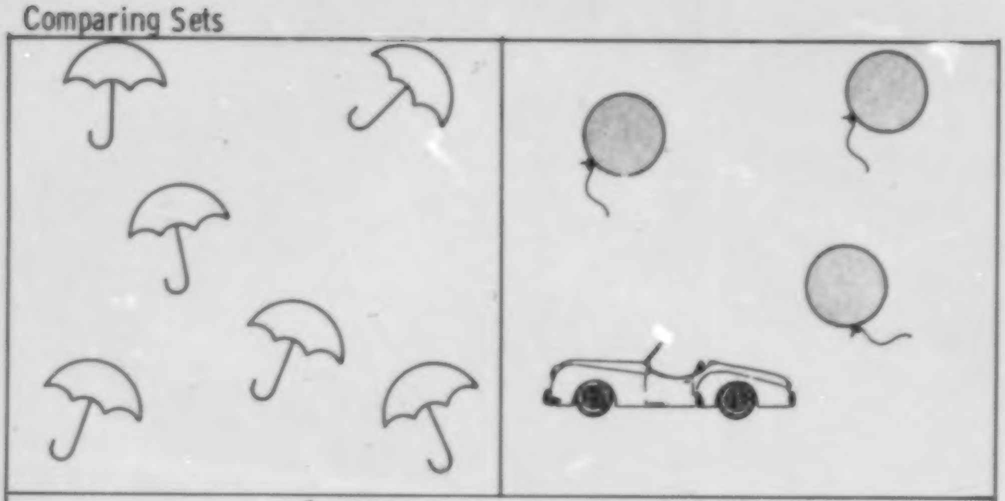


How many? 5

How many? 4

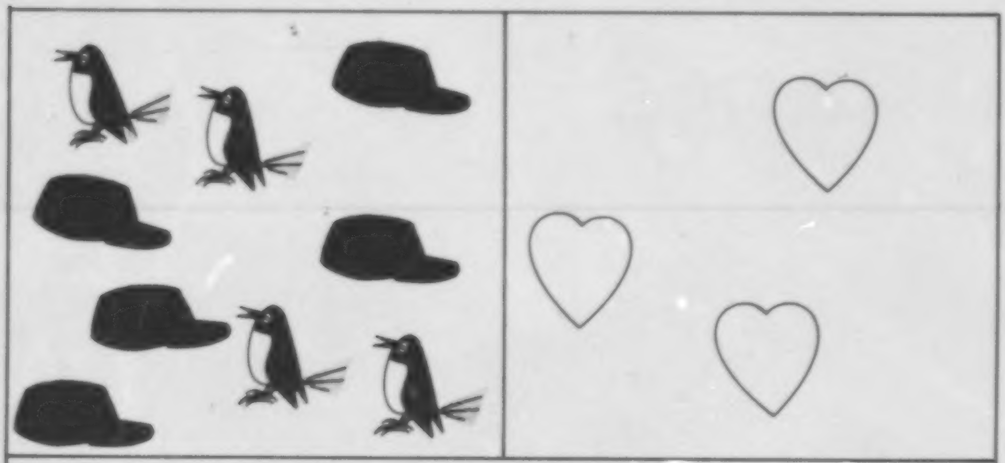
How many more? 1

## Comparing Sets



How many? 6 How many? 4

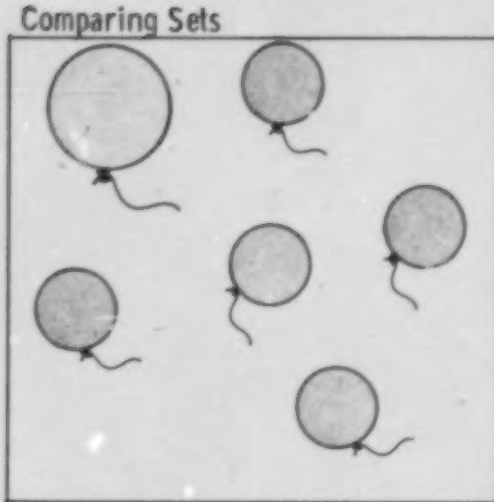
How many more? 2

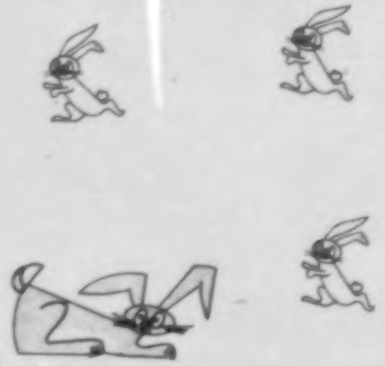


How many? 9 How many? 3

How many more? 6

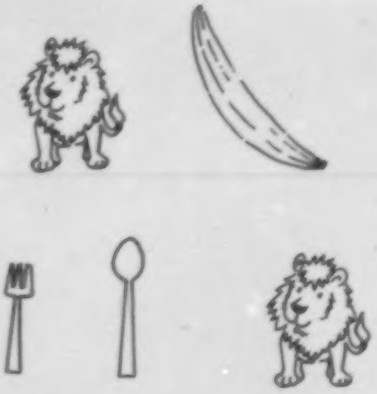
## Comparing Sets





How many? 6 How many? 4

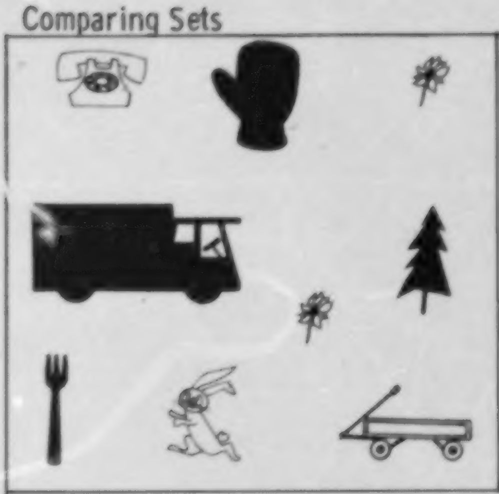
How many more? 2

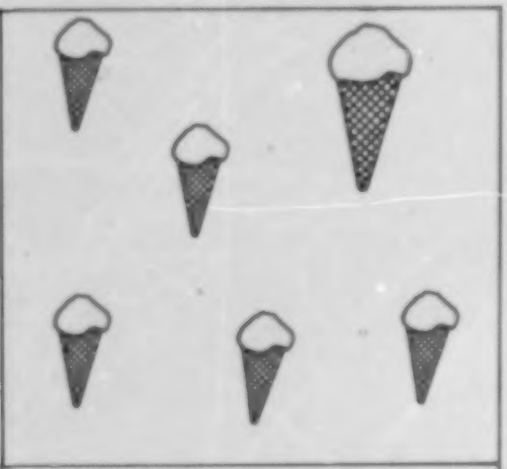


How many? 5 How many? 0

How many more? 5

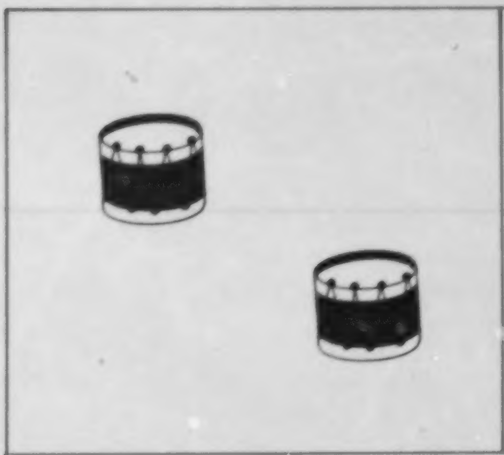
## Comparing Sets

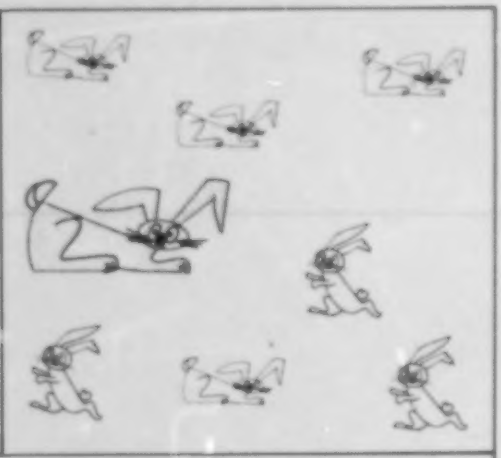




How many? 9 How many? 6

How many more? 3





How many? 2 How many? 8

How many more? 6

- You should follow the work on pages 209-212 by asking how children compared the sets. If no one suggests writing the equation rather than pairing the members of the sets, consider each set of exercises and develop the equations. Hopefully, the children will be able to see that it is not necessary to use the second set if its number is known. It is sufficient to remove a subset which is equivalent to the second set from the set with more members. Consequently, for the first exercise on page 209, we can write 6 - 4 = 2.

If this seems difficult, provide each child with set objects. Use the technique suggested below and word problems such as:

*Mary has 6 apples.*
*Jane has 4.*
*Which girl has more apples?*
*How many more?*

Ask the children to show a set of 6 objects.

*Does Jane have a set of apples that is equivalent to this set?* (No.)
*How many apples does Jane have?* (4.)

Ask the children to remove a subset that is equivalent to the set of apples Jane has.

*How many members in the remaining set?* (2.)
*Does this tell you how many more apples Mary has?* (Yes.)
*Does Mary have 6 - 4 or 2 more apples than Jane?* (Yes.)
*What is the equation?* (6 - 4 = 2.)
*We can write a subtraction equation that describes word problems that ask, "How many more?"*

Continue in the same way with several other similar word problems.

### • PROBLEM SOLVING

Give each child some small objects to work with at his desk. Ask the children to select sets of objects equivalent to objects in the story problems. Notice that the problems differ in the kind of set operation--joining, removing, partitioning. In these problems, children should be expected to find the number to be used in answering the question and then answer the question. An equation is not expected.

Tom has 6 marbles.
Joe has only 4 marbles.
How many more marbles has Tom than Joe?

Jane has 3 crayons.
Then Mary gave her 5 crayons.
How many crayons does Jane have now?

Bill had 7 cars.
Jim took 3 of them.
How many cars does Bill have?

Ann has 3 dolls.
Alice has 8.
How many more dolls does Alice have?

## 7-5. PROBLEM SOLVING AND EQUATIONS

**OBJECTIVE:** To use addition and subtraction to solve simple word problems, and to find missing numbers in equations involving addition and subtraction.

**VOCABULARY:** (No new words.)

**MATERIALS:** Sets of small objects.

**BACKGROUND NOTE:**

There are several different sorts of problems which lead to addition and subtraction equations.

For example:

One set has 4 members and a second set has 9 members. How many members must I join to the first to get a set equivalent to the second? ($4 + \square = 9$ or $9 - 4 = \square$ .)

A set with 4 members is joined to a second set. If the set obtained by joining the two sets has 6 members, how many members did the first set have? ($\square + 4 = 6$ or $6 - 4 = \square$ .)

A set has 3 members. How many members must be joined to the set to get a set of 8 members? ($3 + \square = 8$ or $8 - 3 = \square$ .)

A set has 7 members. How many members must be removed to get a set of 2? ($7 - \square = 2$ or $2 + \square = 7$.)

A set has 3 members remaining after a subset of 4 has been removed. How many members in the original set? ($\square - 4 = 3$ or $3 + 4 = \square$.)

This lesson is devoted to these kinds of problems and the equations which describe them. You should be careful not to classify problems by type, and to encourage each child to use his own method of thinking.

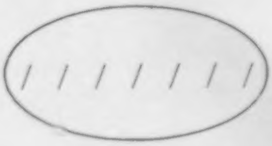
### SUGGESTED PROCEDURE

**PRE-BOOK ACTIVITY:**

Provide each child with set objects. Write an equation such as $3 + 4 = \square$ on the chalkboard. Ask the children to use the objects to help them find the sum. Ask a child to explain what he did. Then continue with equations like $\square + 4 = 9$ and $3 + \square = 8$. Give children an opportunity to explain how they used objects to aid them in completing the equation.

It is important for children to realize that if they know the number of members in a set and also the number of members in a subset, either a set partition or the removal of the known subset will enable them to complete the equation. Follow with an equation such as $9 - 4 = \square$. Hopefully, children will generalize that either of the two procedures previously discussed can be used to complete this equation. If not, ask the children how they would use objects to describe this equation. If removal of a subset is suggested, demonstrate this procedure on the chalkboard. Then ask if a set partition could be used instead. If some of the children agree, ask one of them to show the set partition on the chalkboard.

Then consider an equation like $7 - \square = 3$. Draw a set of 7 members on the chalkboard.



*If you know how many members are in the remaining set, do you know how to partition this set?* (Draw the partition mark so you show a subset of 3.)

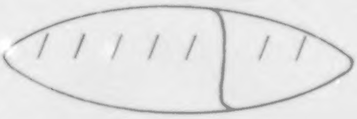
After doing this, ask if the children now know the number of members in the other subset. Ask a child to complete the equation.

Place similar equations on the chalkboard and ask children to use set objects to show the partition.

Finally, consider equations like $\square - 5 = 2$.

*Do you know how many members are in the set?* (No.)
*Do you know how many members are in the subset to be removed?* (5.)
*Do you know how many members are in the remaining set?* (2.)

Ask children to use objects to represent these two subsets. Then make the drawing below on the chalkboard.



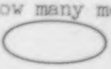
What set partition have I represented on the chalkboard? (A subset of 5 and a subset of 2.)

How many members are in the set? (5 + 2 or 7.)

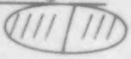
What numeral should I write in the frame? (7.)

Repeat the same procedure with several other equations of this same type.

USING THE PUPIL'S BOOK, page 213:

Call the children's attention to the equations in the upper left hand corner. Ask the children how many members are in the set? (7.) Mark 7 tallies, ///////, in the .

How many elements are in the remaining set? (3.)

How can we partition this set?  .

Complete the equation 7 - 4 = 3. Children may now work independently on the page.

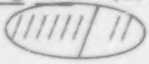
Page 214:

Call the children's attention to the equation in the upper left hand corner. Ask,

Do you know how many members are in the set? (No.)

Do you know how many members are in the subset to be removed? (Yes; 6.)

Do you know how many members are in the remaining set? (2.)

Let us represent these two sets in the ring. (  .)

How many are in the set? (8.)

Write 8 in the frame. Children work independently on the other problems.

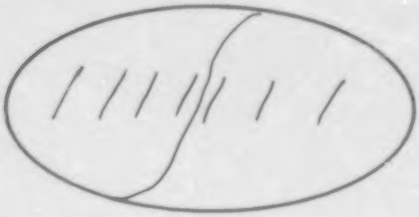
Pages 215-217:

Give the children sets of objects to help them complete the equations. Do several of the problems with the children.

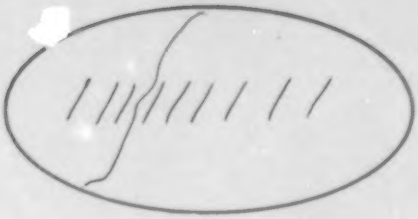
Page 218:

Here children are asked to write + or - in the frames. Do the first three or four exercises so the children understand what is to be done.

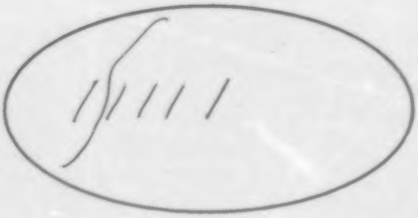
Show the partition.



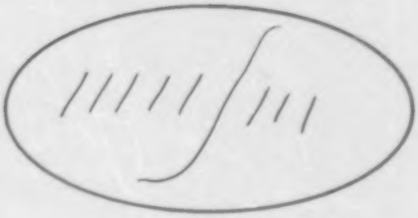
$7 - \boxed{4} = 3$



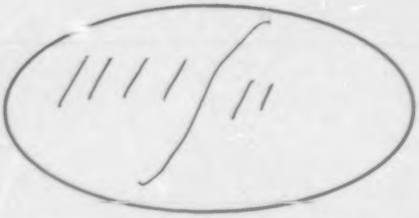
$9 - \boxed{3} = 6$



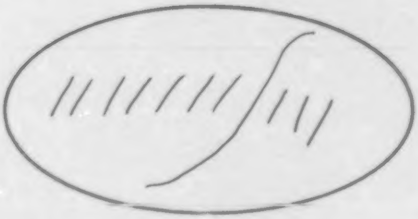
$5 - \boxed{1} = 4$



$8 - \boxed{5} = 3$

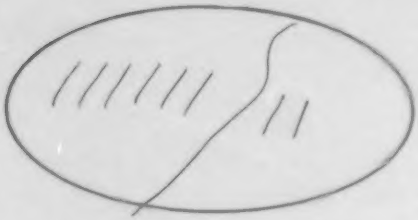


$6 - \boxed{4} = 2$

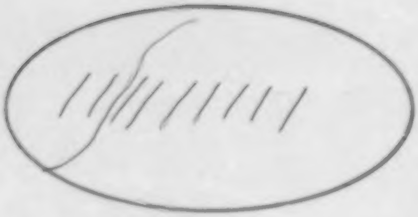


$10 - \boxed{7} = 3$

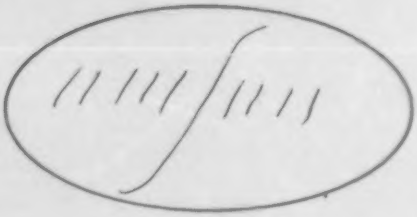
Show the partition.



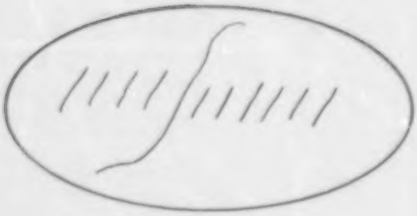
$\boxed{8} - 6 = 2$



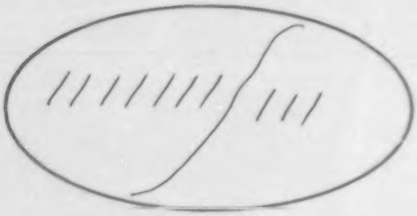
$\boxed{9} - 2 = 7$



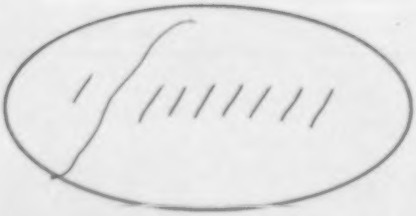
$\boxed{9} - 5 = 4$



$\boxed{10} - 4 = 6$



$\boxed{10} - 7 = 3$



$\boxed{8} - 1 = 7$

Complete the equations.

$\boxed{2} + 5 = 7$

$10 - \boxed{2} = 8$

$7 - 0 = \boxed{7}$

$1 + 9 = \boxed{10}$

$0 + 8 = \boxed{8}$

$\boxed{4} - 1 = 3$

$9 - \boxed{3} = 6$

$8 - 5 = \boxed{3}$

$6 + \boxed{2} = 8$

$\boxed{9} - 3 = 6$

$6 + 2 = \boxed{8}$

$4 - 1 = \boxed{3}$

$5 - \boxed{1} = 4$

$0 + \boxed{7} = 7$

Complete the equations.

$\boxed{8} - 2 = 6$

$3 - \boxed{2} = 1$

$7 - 2 = \boxed{5}$

$8 - 3 = \boxed{5}$

$4 - 0 = \boxed{4}$

$\boxed{7} - 6 = 1$

$10 - 2 = \boxed{8}$

$9 - \boxed{5} = 4$

$\boxed{10} - 4 = 6$

$7 - 4 = \boxed{3}$

$8 - 1 = \boxed{7}$

$10 - \boxed{5} = 5$

Complete the equations.

$3 + \boxed{4} = 7$

$4 + \boxed{5} = 9$

$\boxed{2} + 8 = 10$

$\boxed{8} + 1 = 9$

$7 + 0 = \boxed{7}$

$6 + \boxed{2} = 8$

$\boxed{2} + 4 = 6$

$0 + \boxed{6} = 6$

$5 + 4 = \boxed{9}$

$\boxed{5} + 2 = 7$

$\boxed{4} + 4 = 8$

$\boxed{4} + 6 = 10$

$2 + \boxed{7} = 9$

$7 + 3 = \boxed{10}$

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## Equations

Write + or −

2 + 1 = 3

5 − 3 = 2

7 − 1 = 6

4 + 4 = 8

1 + 9 = 10

4 − 3 = 1

5 + 2 = 7

8 − 5 = 3

9 − 8 = 1

9 − 9 = 0

3 + 4 = 7

1 + 8 = 9

6 − 3 = 3

5 + 4 = 9

7 − 0 = 7
(or +)

3 + 6 = 9

6 − 4 = 2

1 + 5 = 6

- Next present word problems, to find equations which describe the problems, and to name the missing number. Continue to have the children use sets of objects to answer the questions.

The following are word problems you may use.

> There are 9 saucers and 4 cups on the table. How many more cups do we need if we want to put a cup on each saucer? ($4 + \square = 9$, or $9 - 4 = \square$.)
>
> John has 4 cents and Sue has 7 cents. How much must John save to have as many cents as Sue? ($4 + \square = 7$ or $7 - 4 = \square$ .)
>
> Mary had two ribbons. Her mother gave her some ribbons. Now she has seven. How many did her mother give her? ($2 + \square = 7$ or $7 - 2 = \square$ .)
>
> Tom has six boats. He gave some to Bill. Now Tom has 4 boats. How many did he give to Bill? ($6 - \square = 4$ or $4 + \square = 6$.)
>
> Ben has 8 marbles. Three are red. The rest are green. How many are green? ($3 + \square = 8$ or $8 - 3 = \square$ .)
>
> Mark had three cookies. He ate some on the way to school. Now he has only one for lunch. How many did he eat on the way to school? ($3 - \square = 1$ or $\square + 1 = 3$.)
>
> Karen had three caps. Her friend gave her five caps. How many caps does Karen have? ($3 + 5 = \square$ .)
>
> Jack's father gave him three model rockets. His grandfather gave him five. Jack gave one of them to his friend, Douglas. How many rockets does Jack have? ($3 + 5 = \square$ and $8 - 1 = \square$ .)

Display an equation on the chalkboard or flannel board, for example, $7 + \square = 9$. Ask the children to make up a word that goes with the equation.

Ask the children to make up story problems, discuss these, and write equations that go with the stories.

Tell the following story:

Jack had some toy cars.  
Mother gave Jack 2 toy boats.  
Then Jack had 7 cars and boats.  
How many toy cars did Jack have?

Ask the children to show sets equivalent to the sets of toys. Discuss what set operation could be used to find the answer to the question. Ask what equation describes the set operations and have it written on the board. Continue by telling the children the following stories. In each case, have the children tell what equation they can use to help them solve the problem.

1. Father had 3 rakes and 2 shovels. How many rakes and shovels did Father have? (3 + 2 = 5.)
2. Beth had 4 new dresses. Mother bought some more new dresses for Beth. Now Beth has 6 new dresses. How many new dresses did Mother buy for Beth? (4 + 2 = 6 or 6 - 4 = 2.)
3. Jack had some red apples. Mary took 5 of them for Mother. Then Jack had 2 apples. How many red apples did Jack have to begin with? (7 - 5 = 2 or 2 + 5 = 7.)
4. Mary has 7 sticks of gum. Ann has 9. Mary has how many more sticks of gum than Ann? (7 + 2 = 9 or 9 - 7 = 2.)

USING THE PUPIL'S BOOK, pages 219-223:

Read each of the word problems aloud to the children. The pupils should then be directed to complete the page by drawing more objects (rings or X's) as needed or crossing off the subset which is removed. The equation which may be used to help solve the problem is to be written on the line. Then complete the sentence.

## Solving Problems

1. Sam wants 6 toy cars.

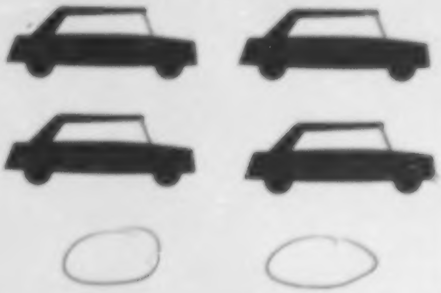
He has 4 toy cars.

How many toy cars must he get?

6 = 4 + 2

Sam must get 2 toy cars.

Problem Picture



2. Pat had 9 marbles.

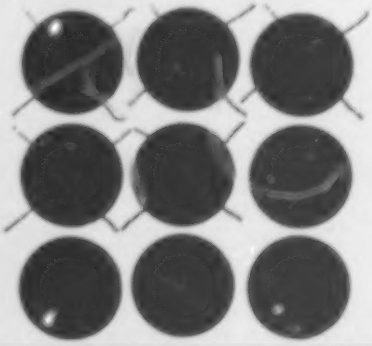
He gave 5 marbles to Dick.

How many marbles did Pat have then?

9 - 5 = 4

Pat had 4 marbles then.

Problem Picture



3. Nan had 6 books.

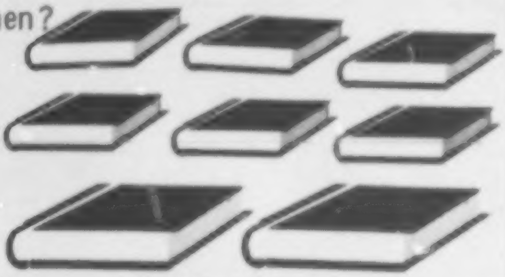
She got 2 new books.

How many books did Nan have then?

6 + 2 = 8

Nan had 8 books then.

Problem Picture



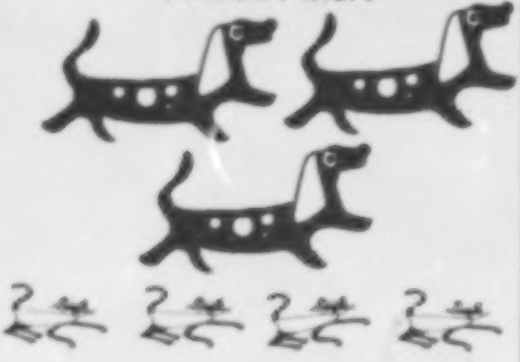
## Solving Problems

1. Bob's dog had 3 puppies.
His cat had 4 kittens.
How many baby animals did
Bob have?

$3 + 4 = 7$

Bob had 7 baby animals.

Problem Picture

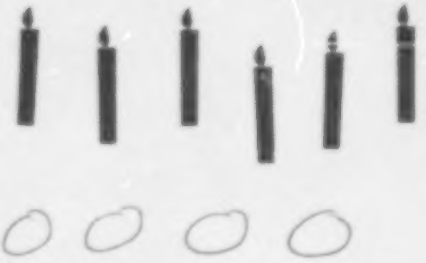


2. Mother needs 10 candles
She has 6 candles.
How many candles must she get?

$10 = 6 + 4$

She must get 4 candles.

Problem Picture

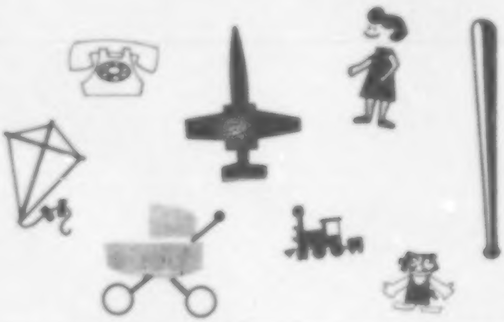


3. Mary had 8 toys.
Tom took 3 of the toys.
How many toys does Mary
have now?

$8 - 3 = 5$

Mary has 5 toys now.

Problem Picture



## Solving Problems

1. David had 6 toy cars.

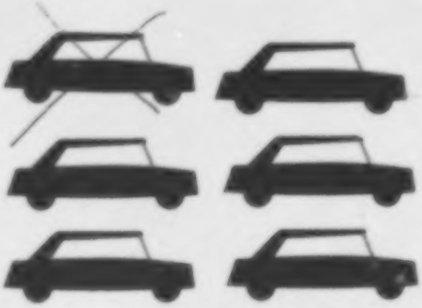
He gave 1 car to Jim.

How many cars does David have?

6 - 1 = 5

David has 5 toy cars now.

**Problem Picture**



2. Sally has 2 crayons.

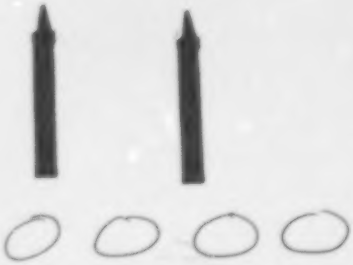
She needs 6 crayons.

How many crayons must she get?

6 = 2 + 4

Sally must get 4 crayons.

**Problem Picture**



3. Joan had 2 cookies.

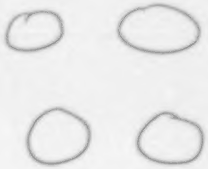
Mother gave 2 more cookies to Joan.

How many cookies did Joan have then?

2 + 2 = 4

Joan had 4 cookies then.

**Problem Picture**

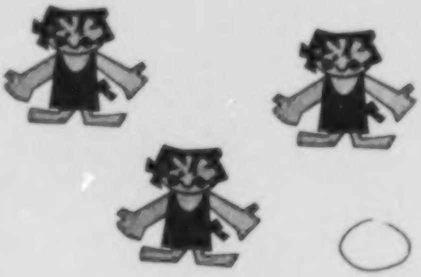


1. Jane wants 4 dolls.
   She has 3 dolls.
   How many dolls must she get?

   4=3+1

   Jane must get 1 doll.

Problem Picture

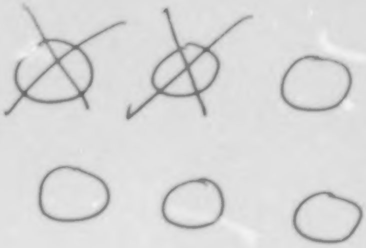


2. Susan had 6 cookies.
   Spot ate 2 cookies.
   How many cookies does Susan have now?

   6-2=4

   Susan has 4 cookies now.

Problem Picture

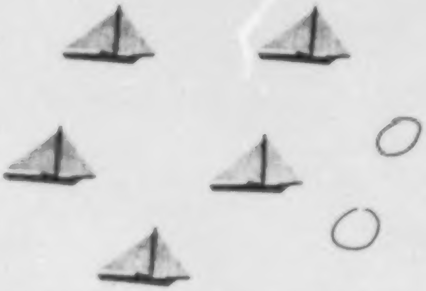


3. Jack had 5 boats.
   He made 2 more boats.
   How many boats did Jack have then?

   5+2=7

   Jack had 7 boats then.

Problem Picture



## Solving Problems

1. Ann made 10 cookies.

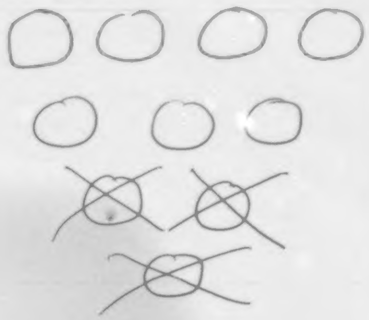
   She gave 3 cookies to Bill.

   How many cookies did Ann have then?

   $10 - 3 = 7$

   Then Ann had 7 cookies.

**Problem Picture**



2. Mrs. Lee had 3 hats.

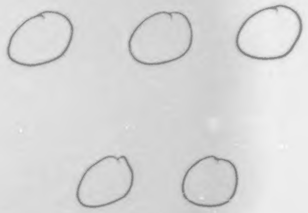
   She got 2 new hats.

   How many hats does she have?

   $3 + 2 = 5$

   Mrs. Lee has 5 hats now.

**Problem Picture**



3. Mother baked some cakes.

   She gave 4 cakes to the church.

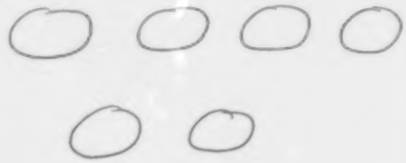
   Then she had 2 cakes.

   How many cakes did Mother bake?

   $4 + 2 = 6$

   Mother baked 6 cakes.

**Problem Picture**



## 7-6. ADDITION AND SUBTRACTION: NUMBERS GREATER THAN TEN

OBJECTIVE: To begin the study of addition and subtraction of numbers greater than ten.

VOCABULARY: (No new words.)

MATERIALS: Sets of small objects for counting.

TEACHING NOTE:

This lesson contains a sample presentation of addition and subtraction of numbers named by 2-digit numerals. The teacher may pursue this as far as seems appropriate with her class.

SUGGESTED PROCEDURE:

> Billy's family is going on a picnic. His mother put ten cookies in the basket. Billy put in thirty more. How many cookies are in the basket?
>
> We can add the number of cookies that Billy put in the basket to the number his mother put in the basket to find the number of cookies. We would write the equation 10 + 30 = _____. To complete the equation, we need to add the numbers. It may help us to think of the number of tens in 10 and in 30.
>
> How many tens are in ten? (1.)
> How many tens in thirty? (3.)
>
> If we add 1 ten and 3 tens, how many tens will we have? (4.) What do we call 4 tens? (Forty.)
>
> When we add 30 to 10, we have 40; we complete the equation 10 + 30 = 40.

If necessary, bundles of ten objects each should be available to use as an aid in solving these problems.

Develop the following problems in the same way:

> Jerry and his friends caught 40 fish on Monday. The next day they caught 10 more. How many fish did they catch in the two days? (40 + 10 = 50.)

Alice had 20 pennies. Her mother gave her 20 more. How many pennies does Alice have now? (20 + 20 = 40.)

Carl had 20 plastic cars and 30 trucks. He took these cars and trucks to Bob's house to play. How many toys did he take? (20 + 30 = 50.)

These problems involve removing a subset:

Steve had 50 marbles. He traded 20 marbles for a kite. How many marbles did he keep? (50 - 20 = 30.)

Ann has 40 dresses for her paper dolls. She left 10 of the dresses at Mary's house. How many dresses does she have to play with at home? (40 - 10 = 30.)

John has 40 pencils. He gave 20 to his brother Jerry. How many pencils does John have now? (40 - 20 = 20.)

- Place two sets of small objects on a table. Do not tell the number of objects in each set.

How can we find how many objects we would have if we joined the sets? (Pupils will suggest that they could join the sets and then count all the objects. This procedure should then be followed.)

Place two new sets of objects on the table. Now tell the children the number of members in each set as you place them on the table. (Be sure that joining the sets of ones will result in a set of not more than 9 ones.)

This set has 22 members.
We will join it to a set with 36 members.
How many objects are in our new set?
Can we find the number of members in our new set without counting each member?
Remember, you know the number of members in each of the sets.

Pursue suggestions that children may give. Hopefully, someone will suggest that since 22 = 2 tens + 2 ones and 36 = 3 tens + 6 ones, we may add the number of ones and then the number of tens. The sum is 58. If no one offers this suggestion, ask:

If we arrange this set of 22 members in sets of tens and ones, how many tens will we have? (2.)
And how many ones? (2.)
If we arrange the set of 36 members in sets of tens and ones, how many sets of ten will we have? (3.)
And how many ones? (6.)

Demonstrate that these answers are correct by arranging the set of 22 and the set of 36 in sets of tens and ones. Join the set of 6 ones to the set of 2 ones.

How many sets of ones do we have now? (8.)

Join the set of 3 tens to the set of 2 tens.

How many tens in all? (5.)
We have 5 tens and 8 ones.
What is the name of the number? (58.)
How many members are in the set? (58.)

Show children an envelope. Tell them that inside is a set of 45 sticks. Show them another envelope. This one has 23 sticks in it. Ask the children if they know the number of sticks we would have if we joined the set of 23 sticks to the set of 45 sticks. If no one suggests adding 23 to 45, offer this idea.

How many tens are in 45? (4.)
And how many ones? (5.)
How many tens are in 23? (2.)
And how many ones? (3.)
If we add 3 ones to 5 ones, what is the number of ones? (8.)

On the chalkboard, write 5 ones + 3 ones = 8 ones.

If we add 2 tens to 4 tens, what is the number of tens? (6.)

On the chalkboard, write 4 tens + 2 tens = 6 tens.

We have 6 tens and 8 ones in all.
How many sticks are in the envelopes? (68.)

Arrange the sets into tens and ones. Join the sets of ones and then the sets of ten to check this work.

- Place a set of objects on the table. Do not tell the number of members.

If I remove 21 members of the set, what will be the number of members remaining? (We will need to count the members of the remaining set in order to find out.)

Place another set on the table. Arrange the set as 4 tens and 8 ones.

This set has 48 members.
Now I will remove a subset of 21 objects.
How many objects are in the set that remains? (Remove 1 one from 8 ones.)
What is the number of ones remaining? (7 ones.) (Remove 2 tens from 4 tens.)
What is the number of tens remaining? (2 tens.)

The remaining set has 2 tens and 7 ones.
What is the number of members in the remaining set? (27.)

Show the children a box. Tell them that inside the box are 57 beads.

If I remove 34 beads, how many beads will be left in the box?

Tell the children we can use numbers to find the number of members in the remaining set. We do this by subtracting 34 from 57.

How many tens are in 57? (5.)
And how many ones are left? (7.)

How many tens are in 34? (3.)
And how many ones? (4.)
If we subtract 4 ones from 7 ones, how many ones are left? (3.)

On the chalkbaord write 7 ones - 4 ones = 3 ones.

If we subtract 3 tens from 5 tens, how many tens are left? (2.)

On the chalkboard write 5 tens - 3 tens = 2 tens.

We have 2 tens and 3 ones in the remaining set if we have subtracted the numbers correctly.
How many beads are in the remaining set? (23.)

Remove the set of 34 beads from the set of 57 beads. Count the number of members in the remaining set in order to check this work.

## Chapter 8

## ARRAYS AND MULTIPLICATION

### BACKGROUND:

A rectangular array is an arrangement of objects into rows and columns. The objects in an array are called its members. Shown below is an array of 3 rows and 5 columns. Note that each of the 3 rows has the same number of members (5) and similarly, each of the 5 columns has the same number of members (3).

x x x x x

x x x x x

x x x x x

The objects need not be all alike. Here, for instance, is an array of 2 rows of 3 members each, all different.

The rectangular arrangements of flannel board objects, blocks on the floor, drawers in a cabinet, panes in a window, compartments in a carton, etc., may all be described as arrays. If an arrangement of objects in rows does not have the same number of members in each row, then it is not called an array.

Multiplication is associated with arrays. When we multiply 5 by 3, for instance, the result is associated with the number of members in an array of 3 rows of 5 members each. In counting the number of members in an array of, for example, 3 rows of 5 members each, children are led to count by rows (5, 10, 15) and to say, "Three fives are fifteen." Later there is a transition to the statement "Three times five equals fifteen" and to the equation $3 \times 5 = 15$. However, it should be noted here that no mastery of multiplication facts is expected in this grade.

Two simple properties of multiplication are pointed out. The first of these is that multiplying numbers in either order always gives the same result. For instance,

$$4 \times 5 = 5 \times 4.$$

Arrays make this very easy to see: when we turn on end an array of 4 rows of 5 members each, we get an array of 5 rows of 4 members each.

X X X X X
X X X X X
X X X X X
X X X X X

× × × ×
× × × ×
× × × ×
× × × ×
× × × ×

By considering an array of just 1 row of, say, 3 members, we see that

$$1 \times 3 = 3.$$

Similarly, 1 multiplied by any whole number is that whole number. Also, by considering an array of 3 rows of just 1 member each, we see that

$$3 \times 1 = 3,$$

and similarly, that any whole number multiplied by 1 is that whole number.

## 8-1. ARRAYS

**OBJECTIVE:** To introduce the array as a means of providing readiness for the concept of multiplication.

**VOCABULARY:** Array, row.

**MATERIALS:** Counting disks, buttons, beans, or other small objects; felt cut-outs for flannel board; hundreds-square paper.

### SUGGESTED PROCEDURE

### PRE BOOK-ACTIVITY:

Give each child a set of 20 counting disks or other small objects. Ask 6 children to select 2 of their disks to put into a box in which you will collect them. When you have collected the disks, discuss with the class the fact that you have joined 6 sets, and that the sets were equivalent to each other, each having 2 members. Have a child count the disks in the box to see how many disks are in the union.

O O
O O
O O
O O
O O
O O

Suggest that the disks be arranged in an array, with 2 in each row, and show that in this arrangement it is possible to count by twos. Explain that this kind of arrangement is called an *array*. It has 6 rows with 2 members in each row. There are 6 sets of 2, and hence 6 twos, or 12 members in the array. Explain that the array we will use is an arrangement of things in rows in which each row has the same number of members and that each member of one row is placed below a member of the preceding row. Point to arrays of window panes, bulletin board pictures, etc., or draw on the chalkboard pictures of arrays of different kinds: 5 rows of 4 members each, 3 rows of 8 members each, 6 rows of 5 members each, etc., and have children observe how they know they are arrays. Ask children to give other examples. Point out that unless all rows have the same number of members the arrangement is *not* called an array. Give examples of arrangements that are not arrays.

Return the disks to the children and ask them to make an array (demonstrate again, if necessary) of 4 rows with 2 members in each row. Ask how many members there are in the array. Continue, in the same way, having other arrays made with 2 members in each row.

Have children make arrays of 6 rows with 2 members in each row, again, and then rearrange the disks to show 2 rows of 6 members each. Children should be aware that these are different arrays, but that they have the same number of members.

Repeat, using an array with 4 rows of 2 members each, and rearranging to form an array with 2 rows of 4 members each.

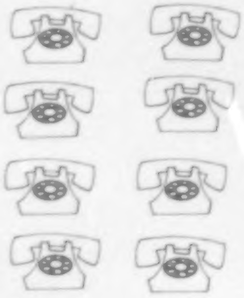
Have children form an array of 3 rows with 5 members in each row. Ask how they might count to find the number of members in the array. (5, 10, 15.) Draw arrays with 5 or 10 members in each row and have children count by rows to find the number of members in the array. Distribute hundreds-square paper. Show children how to make rings or balls in the spaces to form various arrays of 2, 3, 5, or 10 members in each row, as you direct them. For instance, use <u>red</u> crayon to show an array of 8 rows with 2 members in each row. Use <u>blue</u> crayon to show ... Under each array, they should write the number of members in the array.

```
x   x            x   x   x   x   x
x   x            x   x   x   x   x
x   x            x   x   x   x   x
x   x            x   x   x   x   x
x   x
x   x
x   x
x   x
```

<u>USING THE PUPIL'S BOOK</u>, pages 224-226:

Children are to write the number of rows, the number of members in each row, and the number of members in the array. Read each question to the children. Have them write the correct numeral in the blank.

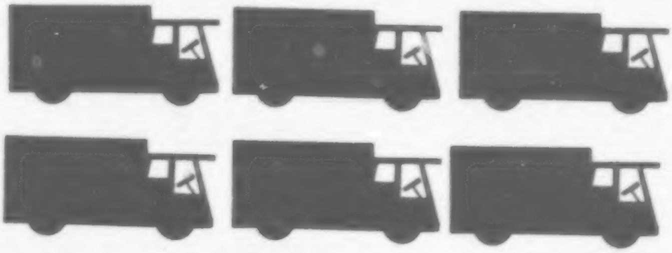
## ARRAYS



How many rows? 4

How many in each row? 2

How many in the array? 8

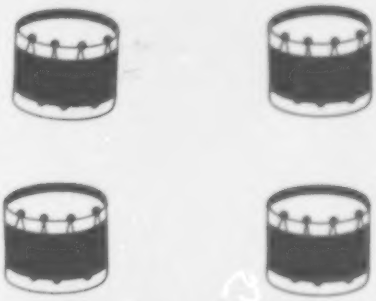


How many rows? 2

How many in each row? 3

How many in the array? 6

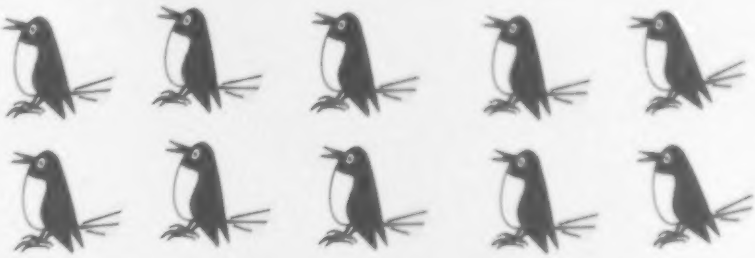
## ARRAYS



How many rows? 2

How many in each row? 2

How many in the array? 4

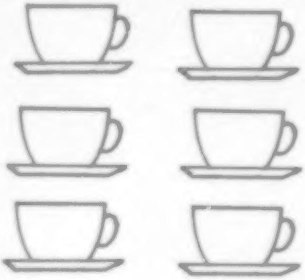


How many rows? 2

How many in each row? 5

How many in the array? 10

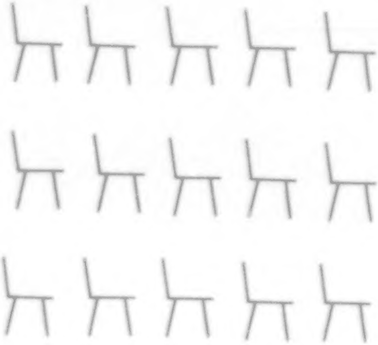
## ARRAYS



How many rows? 3

How many in each row? 2

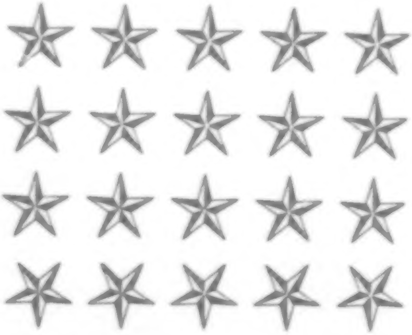
How many in the array? 6



How many rows? 3

How many in each row? 5

How many in the array? 15



How many rows? 4

How many in each row? 5

How many in the array? 20

Show on the flannel board an array of 5 rows with 2 objects in each row. Have children tell how many rows there are and how many objects there are in each row. Point to each row in turn.

Are there 2 objects in this row? (Yes.)

Then ask children how many sets of two there are in the array. (5.) Have them count by twos to find how many members there are in the array. Write:

5 twos are 10.

Show other arrays with rows of 2 objects and repeat, having children tell how many rows, and how many sets of 2 the array has. Have them count by twos. Write:

____ twos are ____.

Include an array with 10 rows with 2 members in each row. When children have said, "Ten twos are 20", show an array with 2 rows of 10 members in each row, and observe that 2 tens are 20.

Have children make arrays with rows of 5 objects and ask how many sets of five there are, etc. Then write:

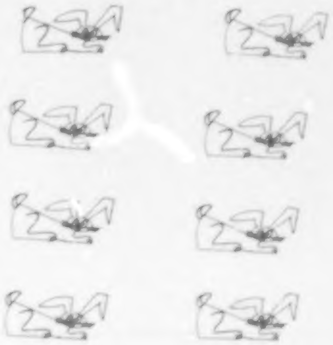
____ fives are ____.

Do the same for rows of 3 members each.

USING THE PUPIL'S BOOK, pages 227-230:

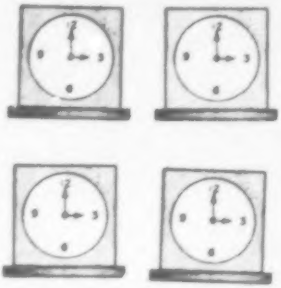
Children first count the rows, then fill in the blanks.

## ARRAYS



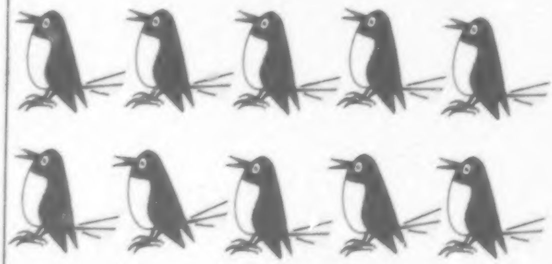
How many sets of 2? 4

4 twos are 8.



How many sets of 2? 2

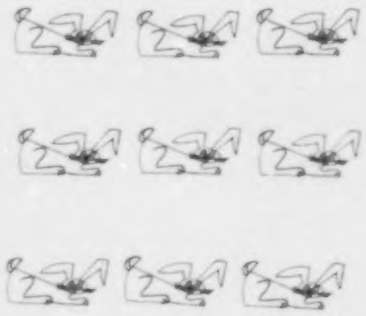
2 twos are 4.



How many sets of 5? 2

2 fives are 10.

## ARRAYS

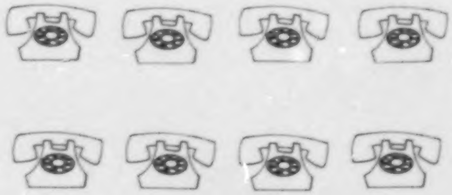


How many sets of 3? 3

3 threes are 9.

How many sets of 10? 3

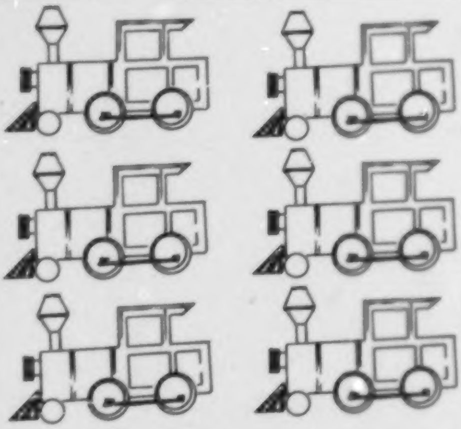
3 tens are 30.



How many sets of 4? 2

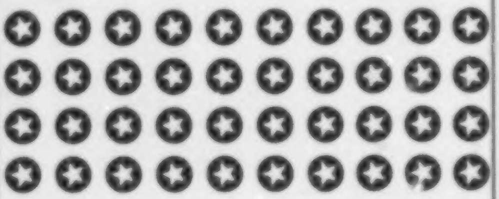
2 fours are 8.

## ARRAYS



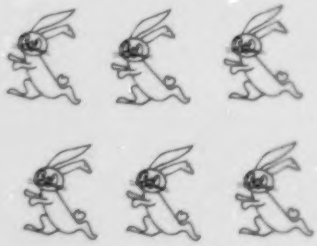
How many sets of 2? 3

3 twos are 6.



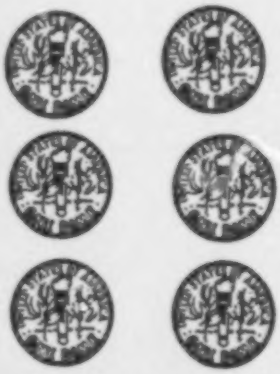
How many sets of 10? 4

4 tens are 40.



How many sets of 3? 2

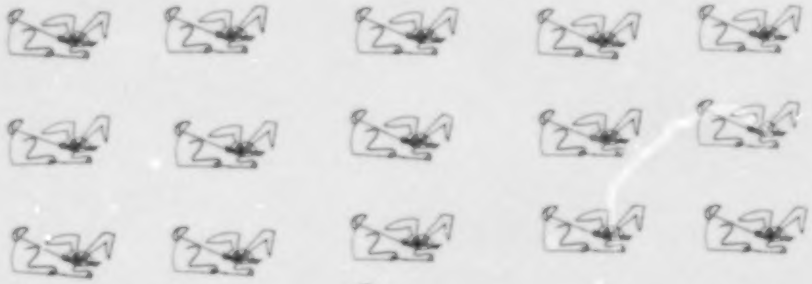
2 threes are 6.



How many sets of 2? 3

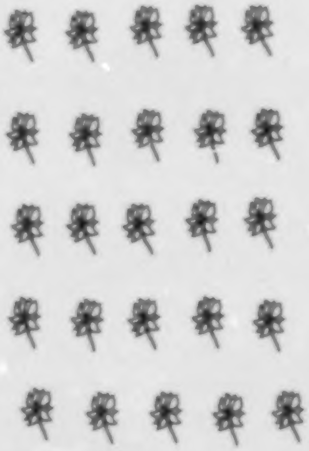
3 twos are 6.

# ARRAYS



How many sets of 5? 3

3 fives are 15.



How many sets of 5? 5

5 fives are 25.

## 8-2. MULTIPLICATION

OBJECTIVE: To introduce the idea of multiplication, using arrays.

VOCABULARY: Multiply, multiplication, times.

SYMBOL: ×

MATERIALS: Felt cut-outs for flannel board; set of counting disks or other small objects; paper bag.

SUGGESTED PROCEDURE

PRE-BOOK ACTIVITIES:

Tell the following story.

Mrs. Brown was getting ready for a picnic.

Eight people were going, and she wanted to take two cookies for each person.

She put the cookies into a box, two at a time.

I'll pretend to be Mrs. Brown and use these circular regions as cookies.

Put the circular regions on the flannel board two at a time. As you do so count: "One person, two people, three people, etc."

How many times did I put 2 disks on the flannel board? (8.)

Is this arrangement an array? (yes.)

How many rows in the array? (8.)

How many members in each row? (2.)

How many members in the array? (16.)

There is an equation that describes this array.

Write:

$8 \times 2 = 16$.

We read this statement: 8 times 2 equals 16.

Does this equation tell us that $8 \times 2$ is another name for 16? (Yes.)

Let's count by twos to be sure that is correct. Two, four, etc.

Provide each child with blocks, disks or other set material. Write $5 \times 2 =$ ____ on the chalkboard. Have the children use set objects to make the array suggested.

How many members in each row? (2.)

How many rows? (5.)

How many times did you place 2 disks on your desk? (5.)

Let's count by twos to find the number of members in the array.

Two, four, ... ten.

What numeral should I write to complete the equation? (10.)
Is 5 times 2 another name for 10? (Yes.)

Continue with the following word problem.

Suppose you earned 5 pennies each school day this week.
That would be Monday, Tuesday, Wednesday, Thursday, and Friday.
How many times would you have earned 5 pennies? (5.)
What is 5 times 5?

Have the children use disks or other objects to form an array and find out. Ask what equation would describe this array. Write the equation on the chalkboard.

Is 5 times 5 another name for 25? (Yes.)

Direct the children to use an array to solve the following problem:

A man has 5 ponies, and they all need new shoes.
He wants to know how many shoes will be needed.
How many shoes for each pony? (4.)
How many ponies are there? (5.)
The number of shoes needed will be equal to 5 times 4.

Write 5 × 4 = ____ on the chalkboard. Have the children use their set objects. They should put 4 disks in a row to show how many shoes the first pony needs, 4 disks in another row for the next pony, etc., to learn that 5 times 4 equals 20. Complete the equation.

Is 5 × 4 another name for 20? (Yes.)
When we say 5 times 4 equals 20, we are MULTIPLYING.
In MULTIPLICATION, we multiply one number by another.

Go directly to problems of the form: what is 3 times 5? Restate the problem in several ways: Three fives are ____? If an array has 3 rows, and each row has 5 members, how many members are in the array? Have children use disks or other objects to help them answer the questions. Write the equation on the chalkboard and complete it when the children have found the product.

USING THE PUPIL'S BOOK, pages 231-233:

Children are to make rings in spaces to form arrays as indicated. They will write the numeral that names the number of members in the array in the blank. Do the first exercise to demonstrate what is to be done.

## Multiplication

Show 3 rows of 5.

| | | | | | | |
|---|---|---|---|---|---|---|
| O | O | O | O | O | | |
| O | O | O | O | O | | |
| O | O | O | O | O | | |
| | | | | | | |
| | | | | | | |

3 × 5 = 15

Show 6 rows of 3.

| | | | | |
|---|---|---|---|---|
| O | O | O | | |
| O | O | O | | |
| O | O | O | | |
| O | O | O | | |
| O | O | O | | |
| O | O | O | | |
| | | | | |

6 × 3 = 18

Multiplication

Show 5 rows of 5.

| | | | | | |
|---|---|---|---|---|---|
| O | O | O | O | O | |
| O | O | O | O | O | |
| O | O | O | O | O | |
| O | O | O | O | O | |
| O | O | O | O | O | |
| | | | | | |
| | | | | | |

5 × 5 = 25

Show 3 rows of 4.

| | | | | | |
|---|---|---|---|---|---|
| O | O | O | O | | |
| O | O | O | O | | |
| O | O | O | O | | |
| | | | | | |
| | | | | | |
| | | | | | |

3 × 4 = 12

## Multiplication

Show 5 rows of 10.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| O | O | O | O | O | O | O | O | O | O |
| O | O | O | O | O | O | O | O | O | O |
| O | O | O | O | O | O | O | O | O | O |
| O | O | O | O | O | O | O | O | O | O |
| O | O | O | O | O | O | O | O | O | O |
| | | | | | | | | | |
| | | | | | | | | | |

5 × 10 = 50

Show 6 rows of 2.

| | | | |
|---|---|---|---|
| O | O | | |
| O | O | | |
| O | O | | |
| O | O | | |
| O | O | | |
| O | O | | |
| | | | |

6 × 2 = 12

FURTHER ACTIVITIES:

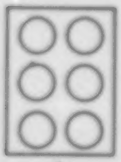
Use word problems such as the following to deepen understanding of multiplication. Read each story to the children. Insist that they use manipulative materials or draw arrays to solve the problem. Write the equation to be completed on the chalkboard. For example, $6 \times 3 =$ _____.

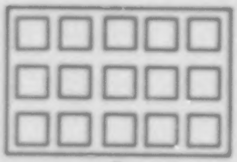
1. Mother washed 5 pairs of stockings. How many stockings did Mother wash?
2. On Mary's street there were 3 houses. In each of the houses lived 3 children. All together how many children lived in the 3 houses?
3. Joe ate 3 apples each day. In 4 days how many apples did Joe eat?
4. Tom hit one home run in each of 3 games. How many home runs did he hit all together?
5. Beth ate 2 cookies each day for 2 days. How many cookies did Beth eat?
6. David put 1 butterfly in each of 5 jars. How many butterflies were in the 5 jars?
7. Each of the girls had 4 dolls. How many dolls would 2 of the girls have?
8. Mark got 3 new books each week for 2 weeks. How many new books did Mark receive?
9. Jim polished 4 pairs of his father's shoes. How many of his father's shoes did Jim polish?
10. Bob had several boxes of toys. Each box had 2 toys in it. How many toys were there in 3 of the boxes?

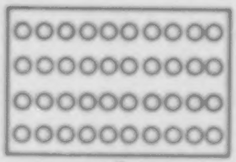
## 8-3. SIMPLE PROPERTIES OF MULTIPLICATION

**OBJECTIVE:** To use arrays to show the commutative property of multiplication and the multiplication property of 1.

**MATERIALS:** Manipulative objects for children, pictures of arrays on tagboard or construction paper as shown:


A


B


C

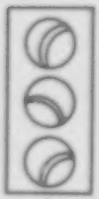

D

Chart on newsprint, as shown:

4 × 5 = 20
8 × 2 = 16
3 × 2 = 6
2 × 5 = 10
1 × 5 = 5

5 × 1 = ___
2 × 3 = ___
5 × 2 = ___
2 × 8 = ___
5 × 4 = ___

### SUGGESTED PROCEDURE:

### PRE-BOOK ACTIVITIES:

Show picture A to the class. Have the array described. (3 rows with 2 members in each row.) Ask children to count by twos to find what 3 times 2 is, and write:

$$3 \times 2 = 6.$$

Turn the picture to show 2 rows of 3 members each.
Have the array described and ask children to count by rows. (3, 6.)
Write:

$$2 \times 3 = 6.$$

Show pictures B and C in the same way. Have children use objects and make arrays of 4 rows of 3, 3 rows of 4, etc., so that they see that exactly the same number of objects are used to make the arrays. Help children to generalize that one number times a second number gives the same result as the second number times the first.

Show picture D. Have the array described (3 rows with 1 member in each row.) Ask how children will count by rows. (By ones.) Write:

$$3 \times 1 = 3.$$

Turn the picture to show the array as 1 row of 3 members. Have the array described, ask children what they will say if they count by rows. (3.) Write the equation: $1 \times 3 = 3$. Draw other arrays either with 1 row or with 1 member in each row, and help children to generalize: any number times 1 is that number, and 1 times any number is that number.

Show the chart and have children read the equations at the left. Read the first one. Ask which equation to the left gives the information needed to complete $5 \times 1 =$ ____. ($1 \times 5 = 5$.) Use crayon or felt pen to write 5 and to draw a line between the sentences that show commutativity. Complete the chart in this way.

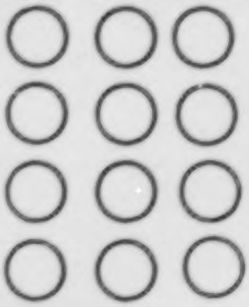
USING THE PUPIL'S BOOK, pages 234-236:

Children are to complete each equation. They should observe that each pair of boxes shows commutativity.

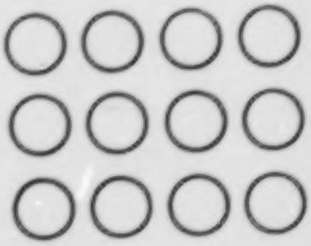
Pages 237-238:

Children should be able to use their understanding of commutativity and of multiplying with 1 as a factor. Provide each child with set objects to aid them in completing the equations on page 238.

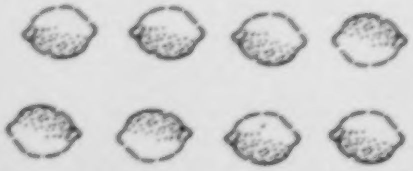
## Multiplication



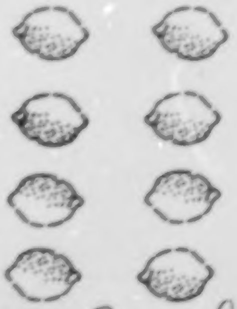
4 × 3 = 12



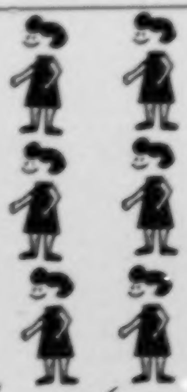
3 × 4 = 12



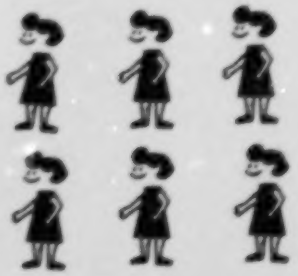
2 × 4 = 8



4 × 2 = 8

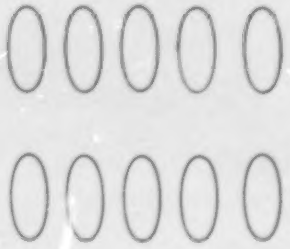


3 × 2 = 6

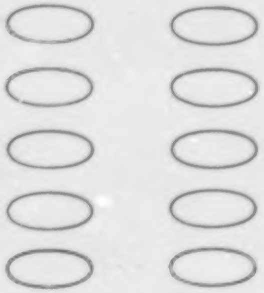


2 × 3 = 6

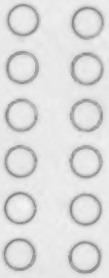
## Multiplication



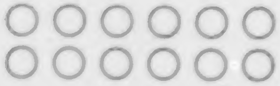
2 × 5 = 10



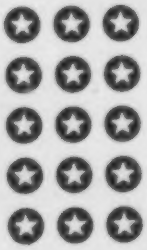
5 × 2 = 10



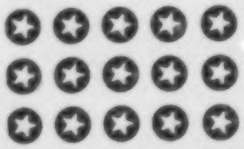
6 × 2 = 12



2 × 6 = 12

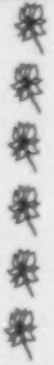


5 × 3 = 15

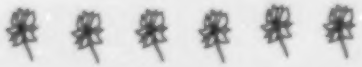


3 × 5 = 15

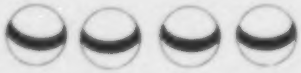
## Multiplication



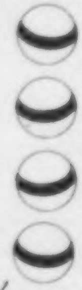
6 × 1 = 6



1 × 6 = 6

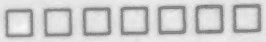


1 × 4 = 4



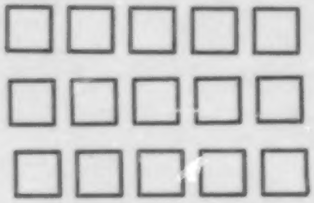
4 × 1 = 4

7 × 1 = 7



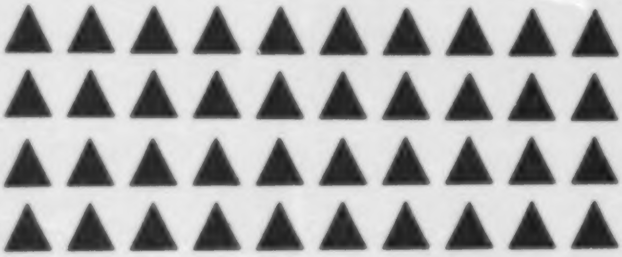
1 × 7 = 7

## Multiplication



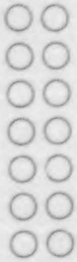
3 × 5 = 15

5 × 3 = 15



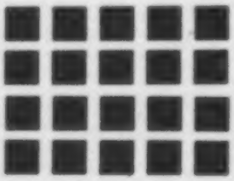
4 × 10 = 40

10 × 4 = 40



2 × 7 = 14

7 × 2 = 14



4 × 5 = 20

5 × 4 = 20

444

## Multiplication

2 × 5 = 10
5 × 2 = 10

3 × 6 = 18
6 × 3 = 18

4 × 5 = 20
5 × 4 = 20

2 × 7 = 14
7 × 2 = 14

1 × 3 = 3
3 × 1 = 3

3 × 7 = 21
7 × 3 = 21

5 × 1 = 5
1 × 5 = 5

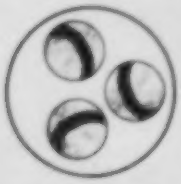
3 × 8 = 24
8 × 3 = 24

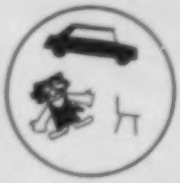
# Chapter 9

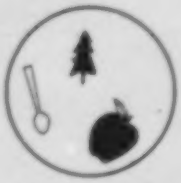
# PARTITIONS AND RATIONAL NUMBERS

## BACKGROUND

Meaning of the whole numbers is developed by observing collections of equivalent sets--sets whose members can be paired so that a one-to-one correspondence is established between the members of any two sets in the collection. The property common to the sets below and to all sets whose members can be placed in a one-to-one correspondence with the members of any one of these sets has the number property, three.







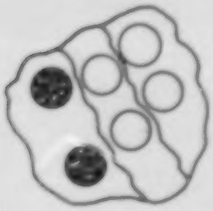
The purpose of this chapter is two-fold. Children learn to partition sets of objects into subsets, and first understandings of the rational numbers of arithmetic are developed by partitioning sets of objects and regions into equivalent parts.

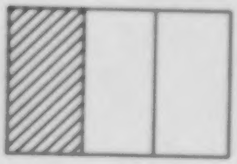
First, sets of objects are partitioned into subsets of a given number of members, and the number of subsets identified. For example, we ask, "How many sets of 5 are there in a set of 20?" We partition a set of 20 into subsets of 5 members each. We note that there are 4 equivalent subsets. We can think of these subsets as forming an array of 4 rows of 5.

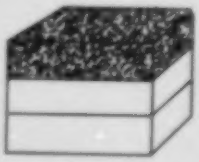
Next, sets of objects are partitioned into a given number of subsets, and then the number of members in each of the equivalent subsets is identified. For example, we ask, "If 20 players are divided into 5 teams with the same number of members on each team, then there are how many members on each team?" Or, "If there are 24 cookies to be shared among 6 children so that each child has the same number of cookies, how many cookies will each child receive?"

Following this, sets of objects are partitioned into equivalent parts and the number of parts are identified. Regions are partitioned into equivalent parts--parts that fit exactly, and the number of parts identified. No number is associated with these parts of a set or region.

Then, physical models are used in developing understanding of the rational numbers, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{2}$, $\frac{2}{3}$, and $\frac{3}{3}$. Consider each of the following as a basic unit:

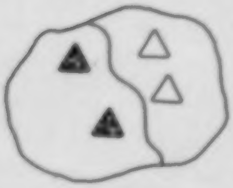


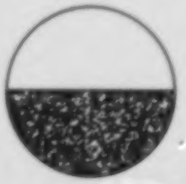


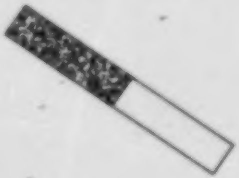


The property common to each of the shaded parts of the unit is the number which can be represented by the fraction, $\frac{1}{3}$, and named "one-third".

Consider each of the following as a basic unit:

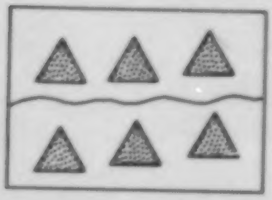




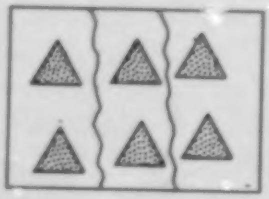


The property common to each of the shaded parts of the unit is the number which can be represented by the fraction, $\frac{1}{2}$, and named "one-half".

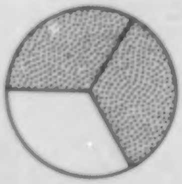
Similar models are used to develop understanding of $\frac{2}{2}$, $\frac{2}{3}$, and $\frac{3}{3}$. For example, the rational numbers associated with the shaded areas of the physical models below (each representing a unit) are



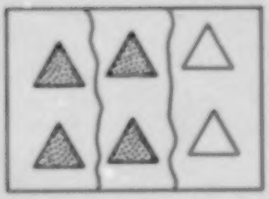
$\frac{2}{2}$



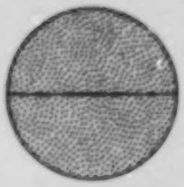
$\frac{3}{3}$



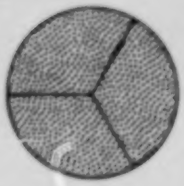
$\frac{2}{3}$



$\frac{2}{3}$



$\frac{2}{2}$



$\frac{3}{3}$

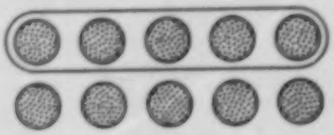
Finally, simple problems such as the following are considered:

> If 10 cookies are distributed fairly to just 2 children, how many cookies will each get? A set of 10 is partitioned into 2 equivalent subsets to find how ~~many members~~ each will have. This leads to the statement,
>
> $$\frac{1}{2} \text{ of } 10 \text{ is } 5.$$

The statement

$$\text{"}\frac{1}{2} \text{ of } 10 \text{ is } 5\text{"}$$

is illustrated by a 2 by 5 array marked in this way:



where we are interested in only one of the two rows and its relation to the entire array.

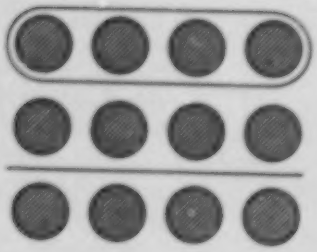
In a similar way we arrive at the statement

$$\frac{1}{3} \text{ of } 12 \text{ is } 4,$$

by asking a question as,

> If 12 cookies are distributed fairly to 3 children, how many cookies will each get?

The number, $\frac{1}{3}$, is associated with the ringed regions in the array if the array is regarded as 1 set. There are 4 objects within this ring.



## 9-1. PARTITIONING SETS INTO EQUIVALENT SUBSETS

**OBJECTIVE:** To partition sets into equivalent subsets of a given number of members.

**VOCABULARY:** (Review) partition, equivalent.

**MATERIALS:** A set of 16 disks or other small objects for each child; A set of 16 disks to be used on the flannel board.

### SUGGESTED PROCEDURE

### PRE BOOK ACTIVITIES:

Place a set of 12 disks (or other small objects) on each child's desk. Ask each child to find out how many objects are on his desk.

Each of you has twelve objects on your desk.
I have twelve disks on the flannel board.
Now pull 4 members of your set to one side of your desk. (You should do likewise with the materials on the flannel board.)
What do we call this part of the set? (Subset.)

Have children continue to pull subsets of 4 members each to other positions on their desks. (As they do this you separate the objects on the flannel board into subsets of 4 members each.) Ask children to make as many subsets with 4 members as they can.

How many subsets do you have? (3.)
Does each subset have 4 members? (Yes.)
Do we have 3 fours in our starting set? (Yes.)
We PARTITIONED our starting set into 3 subsets with 4 members each.

Have children put all disks into one set again and then partition the set into subsets with 3 members each, using the same procedure as above. You follow a similar procedure with the objects on the flannel board. When 4 subsets of 3 have been obtained, ask the following questions:

You partitioned your set of 12 members.
How many subsets do you have? (4.)
How many members in each subset? (3.)
Do you have 4 threes in your starting set? (Yes.)

Use the same procedure and partition the set of 12 into subsets of 2. Say "12 is 6 twos." Partition the set into subsets of 6. Say, "12 is 2 sixes."

USING THE PUPIL'S BOOK, pages 239-240:

Children should first tell how many objects are in the picture. They complete the first sentence. Then, they should ring sets, as indicated, and tell how many equivalent subsets there are.

Pages 241-242:

Not only do children tell how many objects in the picture and the number of equivalent subsets of the given number of members, but they also relate the situation to multiplication language.

FURTHER ACTIVITIES:

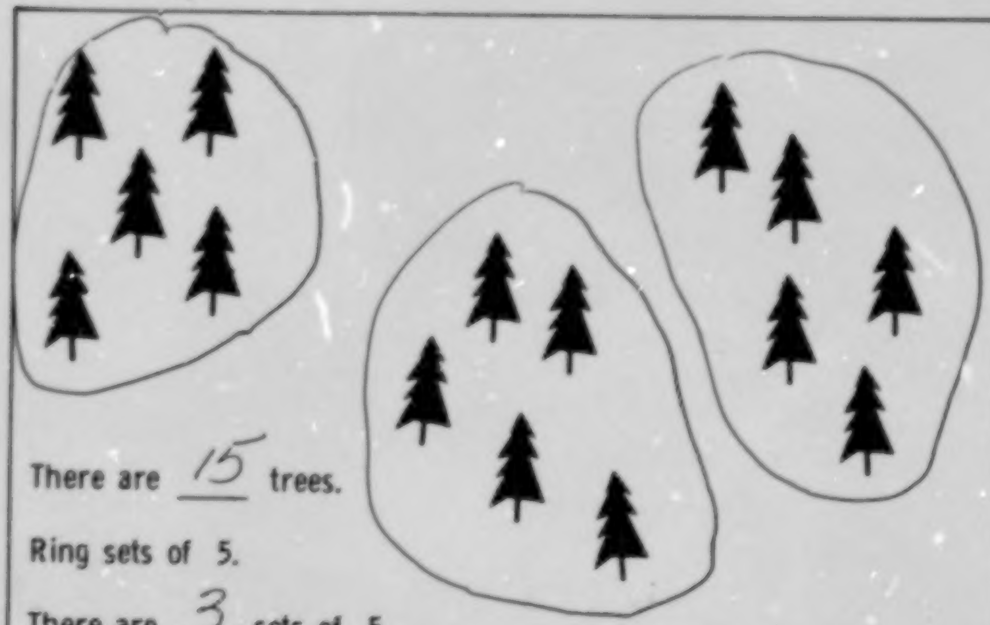
1. Have children put 8 objects on their desks. Let individual children suggest ways of partitioning. ("Bill, how many things shall we use for our first subset?") If no child suggests it, make certain that sometime during the period it is noticed that a set of 8 can be partitioned into subsets of 2 and into subsets of 4. Also, if not suggested, ask children to partition the set into subsets of 1 member each and into subsets of 8 members each.

   What would happen if you partitioned your set into subsets of 1 member each? (Then we would have 8 subsets of 1 member each.)

   What would happen if you partitioned your set into subsets of 8 members each? (Then we would have just 1 subset of 8 members.)

2. Draw sets on chalkboard (balls, trees, kites, etc.) and ask children to draw rings around equivalent subsets of various numbers of members. Use sets of 6, 4, 2, 9, 10, etc. While one child is drawing the rings on the chalkboard, the other children can make a similar set on their desks and show the subsets by moving the objects.

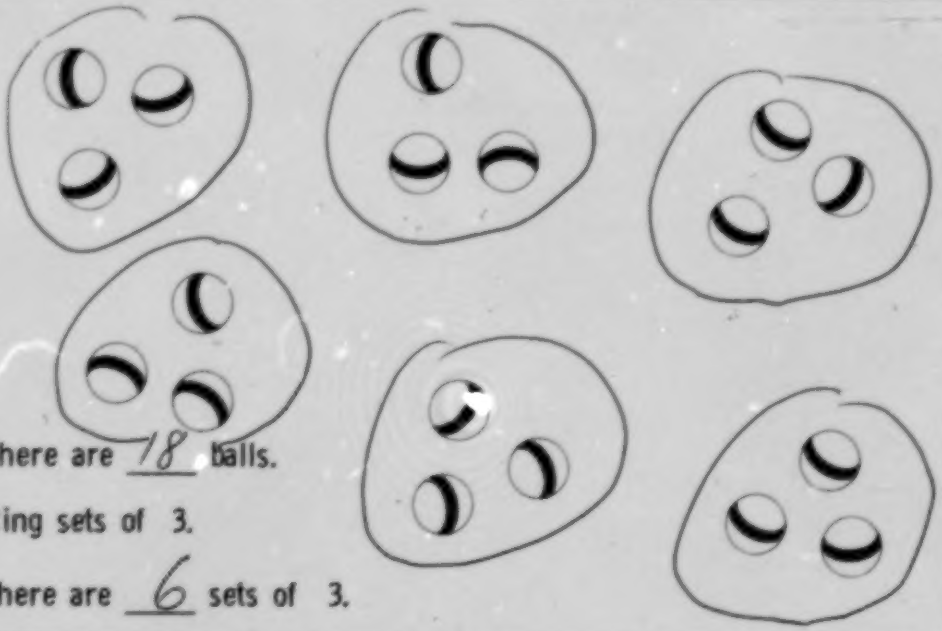
## Partitioning



There are 15 trees.

Ring sets of 5.

There are 3 sets of 5.

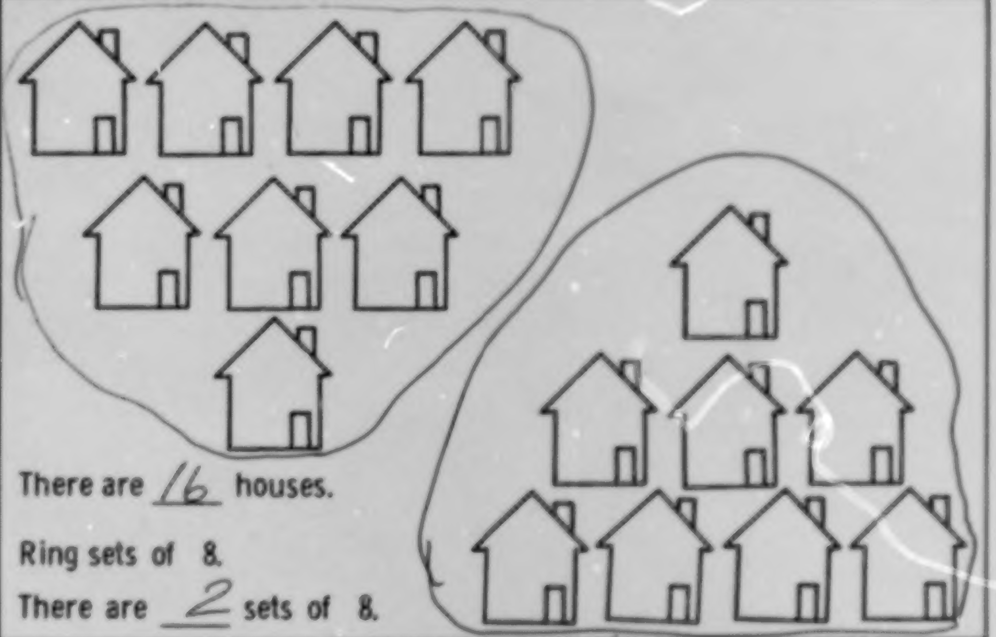


There are 18 balls.

Ring sets of 3.

There are 6 sets of 3.

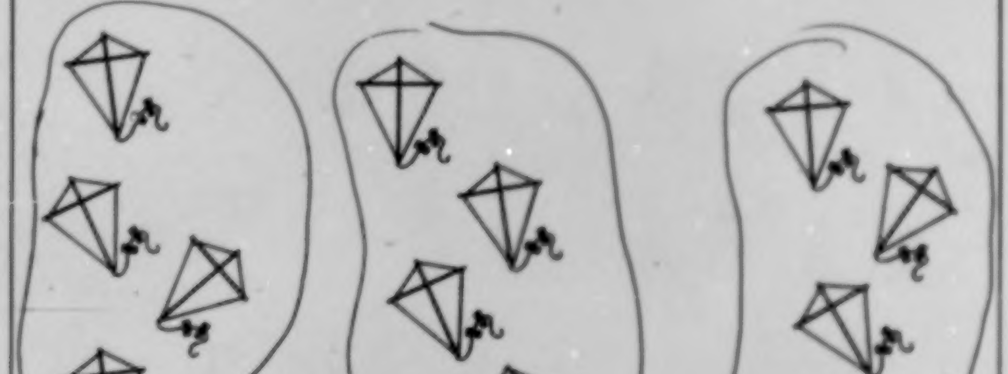
## Partitioning



There are 16 houses.

Ring sets of 8.

There are 2 sets of 8.

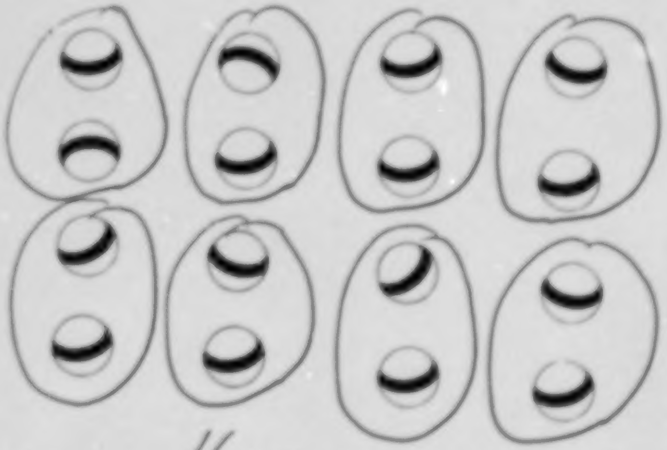


There are 12 kites.

Ring sets of 4.

There are 3 sets of 4.

## Partitioning

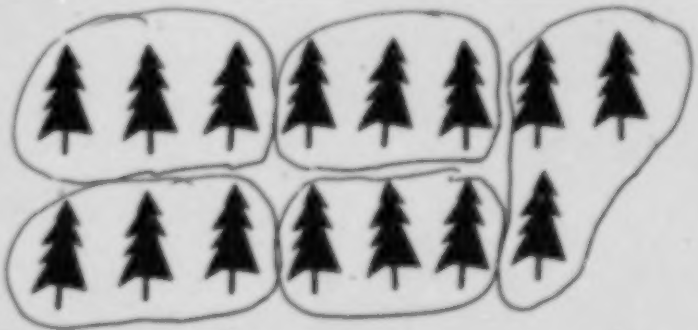


There are 16 balls.

Ring sets of 2.

There are 8 sets of 2.

16 is 8 twos.



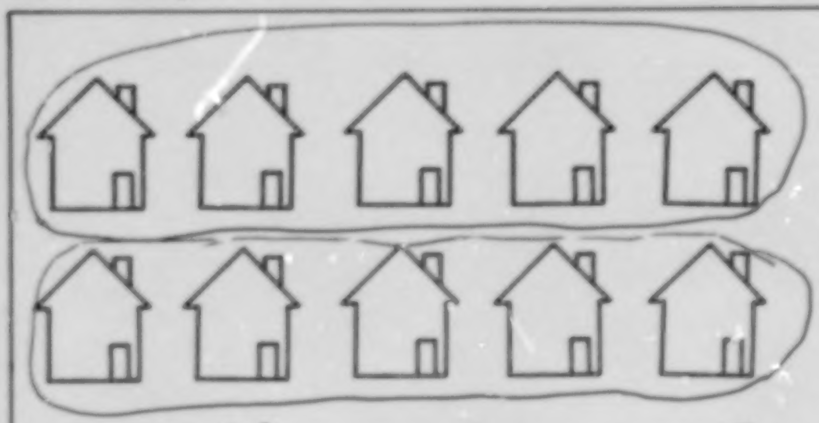
There are 15 trees.

Ring sets of 3.

There are 5 sets of 3.

15 is 5 threes.

## Partitioning

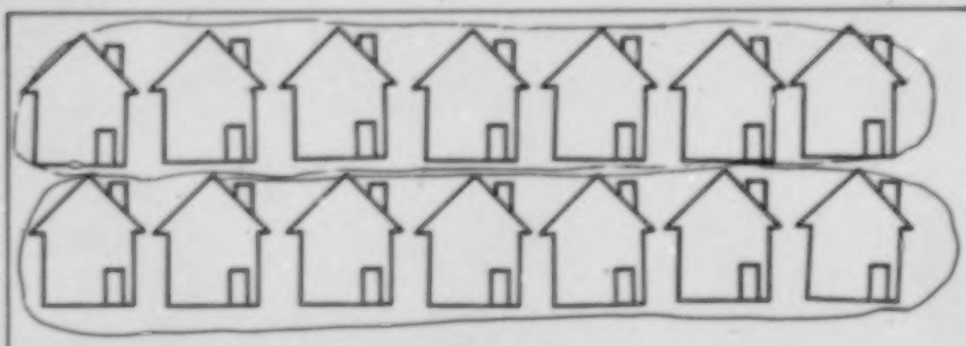


There are 10 houses.

Ring sets of 5.

There are 2 sets of 5.

10 is 2 fives.



There are 14 houses.

Ring sets of 7.

There are 2 sets of 7.

14 is 2 sevens.

Place a set of 16 objects on the flannel board. Ask a child to tell how many objects are on the flannel board. Then ask a child to partition the set into sets of four. Observe that there are 4 sets of four in a set of sixteen.

> We know that there are four members in each of these subsets. How can we arrange the objects so it would be easier to see that there are 4 objects in each set?

If children do not suggest an array, then ask a child to show how these 4 sets of 4 can be arranged in an array.

> We know that if there are 4 in the first ROW and the other rows are the same, then each row has 4 members.
> We can make a set of 16 into an array having 4 rows and 4 members in each row.
> We see that a set of 16 can be partitioned into 4 subsets of 4.
> How many fours are there in 16?
> We see that 16 is 4 fours.

Repeat this procedure, partitioning a set of 16 into subsets of 2 by arranging the objects in an array so that each row has 2 members.

> How many rows do we have? (8.)

Have the set of 16 arranged in rows of 8 members each.

> How many rows do we have? (2.)

Have the set of 16 arranged in rows with 1 member in each row, etc.

Distribute counters (disks, buttons, etc.) Ask children to show an array to find how many sets of 3 there are in a set of 9.

Continuing the same way, have the children find how many rows of 7 members each will be in an array of 14 members. How many rows of 5 members each will be in an array of 10 members? etc.

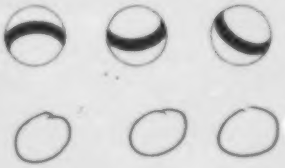
USING THE PUPIL'S BOOK pages 243-244:

Children are to complete the starting set as indicated. They may draw similar objects or use rings or X's. Then they complete the sentences.

## Partitioning

Show 6 balls.

Have 3 balls in each row.



There are 6 balls.

6 is 2 threes.

6 = 2 × 3.

Show 6 balls.

Have 2 balls in each row.

There are 6 balls.

6 is 3 twos.

6 = 3 × 2.

Show 8 balls.

Have 4 balls in each row.

There are 8 balls.

8 is 2 fours.

8 = 2 × 4.

Show 10 balls.

Have 2 balls in each row.

There are 10 balls.

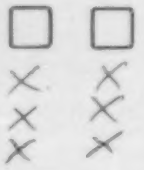
10 is 5 twos.

10 = 5 × 2.

## Partitioning

Show 8 boxes.

Have 2 boxes in each row.



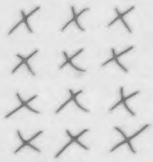
There are 8 boxes.

8 is 4 twos.

8 = 4 × 2.

Show 12 boxes.

Have 3 boxes in each row.



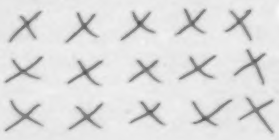
There are 12 boxes.

12 is 4 threes.

12 = 4 × 3.

Show 15 boxes.

Have 5 boxes in each row.



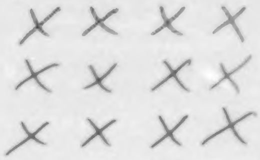
There are 15 boxes

15 is 3 fives.

15 = 3 × 5.

Show 12 boxes.

Have 4 boxes in each row.



There are 12 boxes.

12 is 3 fours.

12 = 3 × 4.

## 9-2. PARTITIONING INTO A GIVEN NUMBER OF EQUIVALENT SETS

**OBJECTIVE:** To partition a set into a given number of equivalent subsets.

**VOCABULARY:** (No new words.)

**MATERIALS:** A set of 16 counters (disks, buttons, etc.) for each child; a set of 16 disks to be used on the flannel board.

### SUGGESTED PROCEDURE

**PRE-BOOK ACTIVITIES:**

Present the following problem:

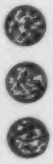
Mary, Sue and Betty have 12 cookies.
They want to share the cookies so that each girl will have the same number of cookies.
How can they do this?

Children will probably suggest giving one to each girl in turn until all cookies have been distributed; "passing them out" until all are gone. Explain that we must see that each girl gets the same number of cookies.

Let these disks represent the cookies.
How many shall we put on the flannel board? (12.)

Place three yarn rings on the flannel board so that each girl's cookies may be placed inside the ring. Remove a subset of 3 from the set of 12 and place a cookie inside each ring. Continue this procedure until no more subsets of 3 can be removed from the starting set. Ask how many cookies each girl will get. Ask children to describe another way of showing the 3 subsets. If they do not suggest putting the disks in rows, suggest that we put the disks in three rows. The first row then shows the cookies that Mary gets, the second row shows Sue's cookies and the third row shows Betty's cookies.

Again, remove a subset of 3 from the starting set of 12. Place the disks as the first member in each of 3 rows, as,



Remove a second subset of 3 and place a disk in each of the 3 rows. Continue until the 4 subsets have been removed.

If children do not observe that an array has been made and that it is easy to see that each row has 4 members, ask them to tell how many rows and how many members in each row.

Let us complete this sentence (which you write on the chalkboard)

12 is three______'s.

Discuss the fact that making an array helps them to find out how many cookies each girl receives. (4.)

Have children use objects on their desks to find out how many members in each set if a set of 10 is partitioned into 2 equivalent subsets. When they find the answer, then write:

10 is two__5__'s.
$10 = 2 \times 5$

Continue with several other problems, such as, a set of 12 as 4 subsets of how many members, a set of 8 as 4 subsets of how many members, a set of 9 as 3 subsets of how many members, a set of 14 as 2 subsets of how many members, etc. To solve these problems and similar problems children should use objects.

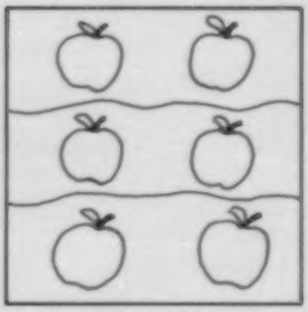
**USING THE PUPIL'S BOOK**, pages 245-247:

Children are to first tell the number of objects in the picture. Then they separate the set into the number of given subsets, drawing a ring around each subset or drawing lines to separate the objects into the subsets, and complete the sentences.

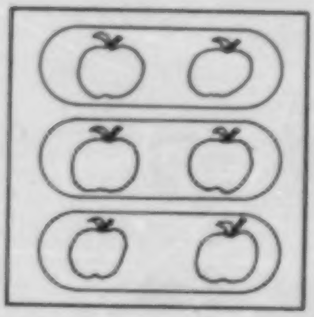
Discuss the first example with the children. For example,

There are 6 apples.
Imagine that three children are to share the apples.
Show the apples each child gets.

Make clear that they are to use all the objects and that the same number of objects must be in each subset. Instruct them as to how they may show the subsets, as,



or



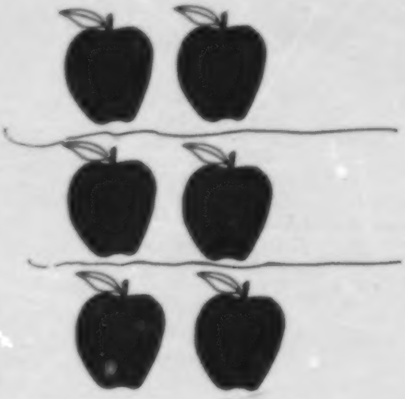
Suggest that they imagine problems for the other pictures, such as,

> There are 6 blocks. Think of putting the same number of blocks in each of 2 boxes. Show the blocks that go in each box.

## Partitioning

There are 6 apples

Show 3 sets.

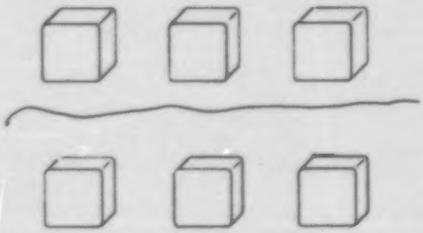


6 is three 2 's.

6 = 3 × 2.

There are 6 blocks.

Show 2 sets.

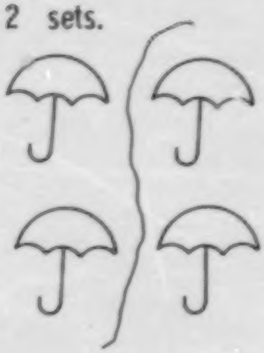


6 is two 3 's.

6 = 2 × 3.

There are 4 umbrellas.

Show 2 sets.

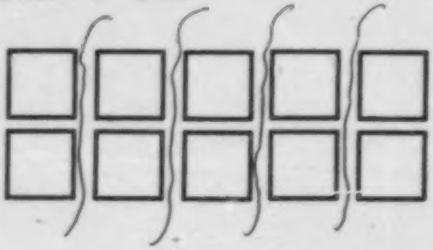


4 is two 2 's.

4 = 2 × 2.

There are 10 squares.

Show 5 sets.



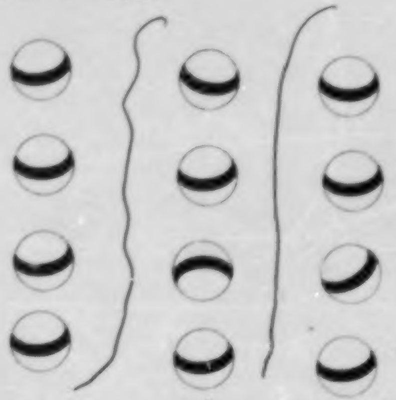
10 is five 2 's.

10 = 5 × 2.

## Partitioning

There are 12 balls.

Show 3 sets.

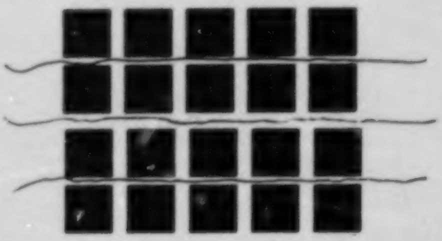


12 is three 4's.

12 = 3 × 4.

There are 20 blocks.

Show 4 sets.

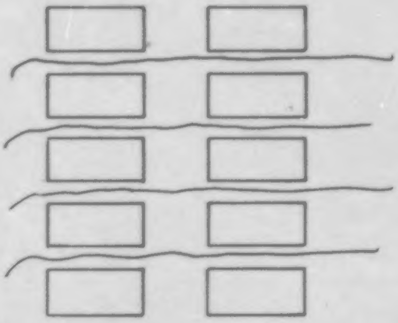


20 is four 5's.

20 = 4 × 5.

There are 10 rectangles.

Show 5 sets.

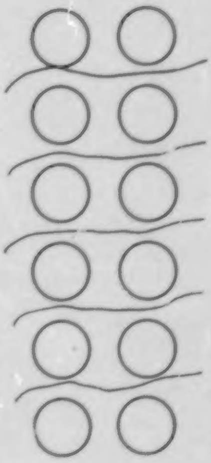


10 is five 2's.

10 = 5 × 2.

There are 12 circles.

Show 6 sets.



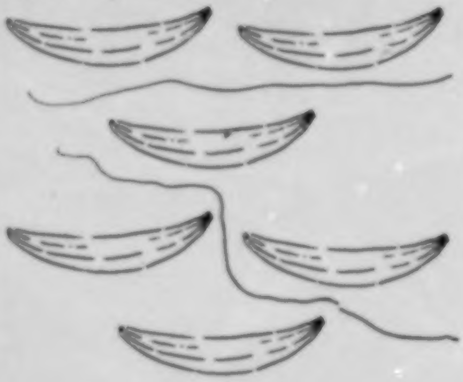
12 is six 2's.

12 = 6 × 2.

## Partitioning

There are 6 bananas.

Show 3 sets.

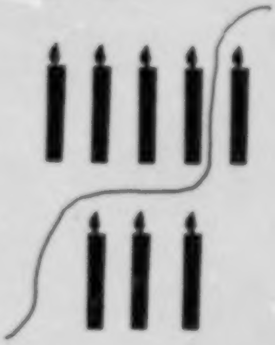


6 is three 2 's.

6 = 3 × 2 .

There are 8 candles.

Show 2 sets.

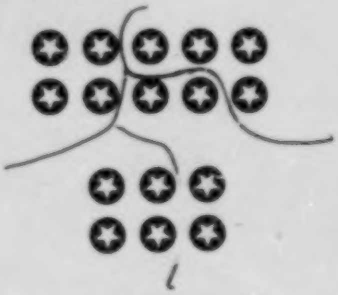


8 is two 4 's.

8 = 2 × 4 .

There are 16 stars.

Show 4 sets.

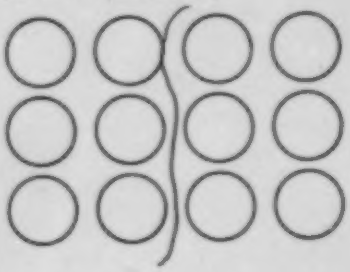


16 is four 4 's.

16 = 4 × 4 .

There are 12 circles.

Show 2 sets.



12 is two _____ 's.

12 = 2 × _____ .

## 9-3. PARTS OF REGIONS AND SETS

**OBJECTIVE:** To partition given regions and sets of objects into two and three congruent or equivalent parts.

**VOCABULARY:** (No new words.)

**MATERIALS:** Sheets of yellow and blue paper; rectangular and circular regions and set of objects to be used on the flannel board; drinking straws; and other objects that can be separated into congruent or equivalent parts. A piece of yarn. A set of 12 objects (disks, buttons, etc.) for each child. A piece of string for each child.

### SUGGESTED PROCEDURE

### PRE-BOOK ACTIVITIES:

Place an "apple" or circular piece of felt on the flannel board.

Here is one apple.
How can we cut it to have two parts the same size? (If children suggest ways, cut the "apple" according to their directions. It may be necessary to take another "apple" and try again if they find that the parts are not the same size.)

Then take one part from the flannel board, holding it in your hand so children can see it. Ask,

How many parts are in my hand? (1.)
How many parts are now on the flannel board? (1.) (Return the part to the flannel board.)
How many equivalent parts in the whole apple? (2.)

Continue the discussion by placing a circular region (or rectangular region) on the flannel board--one which has already been cut into 3 congruent parts.

Here is a piece shaped like a circle.
It has already been cut.
How many pieces are there? (3.)
Are these parts all the same size?
How can you tell? (Place one part on each of the others to see if they fit exactly.)
How many parts in the whole piece? (3.)

Then remove two parts from the flannel board, holding them so that the children can see them.

How many parts do I have in my hand? (2.)

How many parts are still on the flannel board? (1.) (Put the pieces on the flannel board again.)

Now how many pieces are on the board? (3.)

Do we have our starting region on the flannel board? (Yes.)

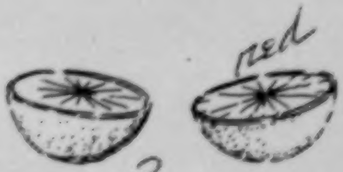
Continue the discussion using drinking straws as the starting units. Tape one to the chalkboard. Observe that the other straws are just as long as the one on the board. Cut one of these into 2 congruent parts. Cut the other into 3 congruent parts. Place these below the straw on the board. Indicating the straw cut into two parts, ask a child to draw a ring around one of the parts. Then erase the ring and ask another child to draw a ring around two of the parts. In a similar way, note the three parts of the other straw--1 part, 2 parts, and 3 parts--all of the same size.

Use other materials which enable children to observe first a whole unit and then to recognize that it has been partitioned into 2 or 3 congruent parts--parts of the same size and shape.

USING THE PUPIL'S BOOK, pages 248-249:

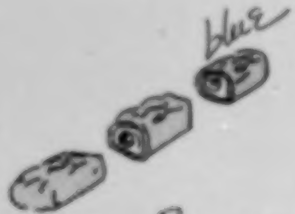
Children observe the number of congruent parts into which the unit has been partitioned. Then they complete the sentence and color the number of parts indicated.

## Parts of Regions



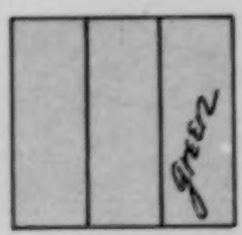
There are 2 parts.

Color 1 part red.



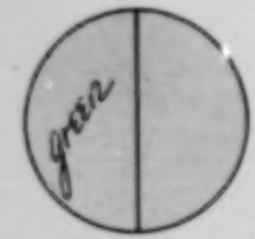
There are 3 parts.

Color 1 part blue.



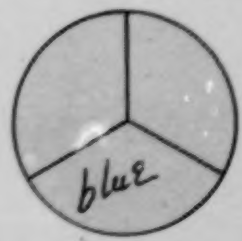
There are 3 parts.

Color 1 part green.



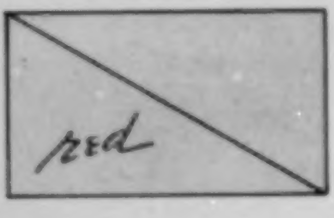
There are 2 parts.

Color 1 part green.



There are 3 parts.

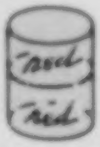
Color 1 part blue.



There are 2 parts.

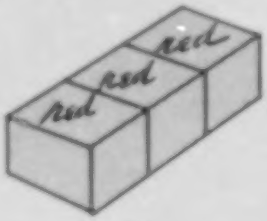
Color 1 part red.

## Parts of Regions



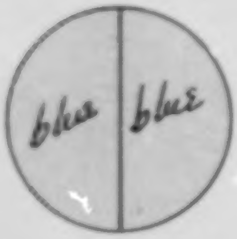
There are 2 parts.

Color 2 parts red.



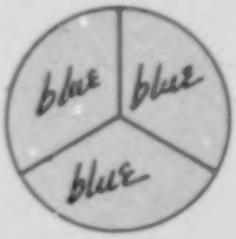
There are 3 parts.

Color 3 parts red.



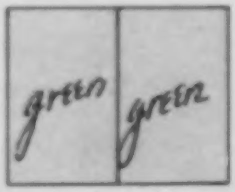
There are 2 parts.

Color 2 parts blue.


blue
blue
blue

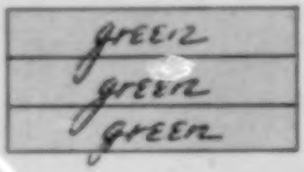
There are 3 parts.

Color 3 parts blue.


green
green

There are 2 parts.

Color 2 parts green.


green
green
green

There are 3 parts.

Color 3 parts green.

Place a set of 6 objects on the flannel board.

Here we have one set of objects.
Let's partition this set into parts, so that the parts are equivalent.

Separate the objects into 2 parts. Place a piece of yarn around each part. Ask a child to show one part of the set. Then ask how many parts do not have rings around them. (1.) Remove the yarn.

How many parts do we have on the board? (2.)
Do we have our starting set? (Yes.)

Then ask a child to separate the set into 3 equivalent parts. Ask how many parts there are. Ask a child to use the yarn to show 1 part of the set. Removing the yarn, ask another child to show 2 parts of the set, again using the yarn to show the two parts. Again, removing the yarn, ask another child to show 3 parts of the set.

Give each child 12 counters (disks, buttons, etc.) and a piece of string. Ask each child to place a set of 4 objects on his desk. When they have done this, ask each to partition his set into 2 equivalent parts. After this task is completed, ask each to use the string to show one part. Then ask each child to [illegible] two parts. (At this time, we are not concerned with the number of objects in each part.)

Continue the lesson by asking each child to put a set of 6 objects on his desk. First partition into 2 equivalent parts and give instructions as above. Then, ask each to partition the set of 6 into 3 equivalent parts. Using their strings, they identify 1 part, 2 parts, and 3 parts.

Use sets of 2, 8, 10, and 12 for partitioning sets into 2 equivalent parts.

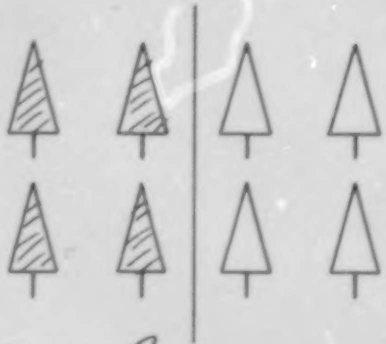
Use sets of 3, 9, and 12 for partitioning sets into 3 equivalent parts.

USING THE PUPIL'S BOOK, pages 250-251:

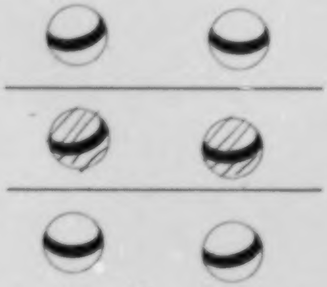
Children observe the number of equivalent parts into which the set has been partitioned. Then they complete the sentence and color each object in the parts indicated. Demonstrate with first exercise on page 250. Alert children to horizontal partitioning in the exercise with six balls. Ask:

Can you find the lines partitioning the balls?

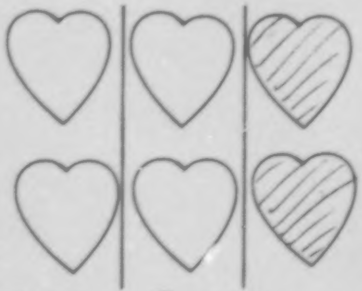
## Part of Sets



There are 2 parts. Color 1 part.

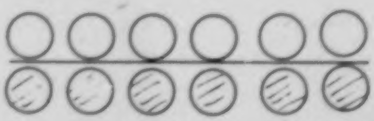


There are 3 parts. Color 1 part.



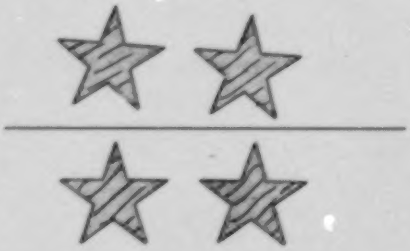
There are 3 parts.

Color 1 part.



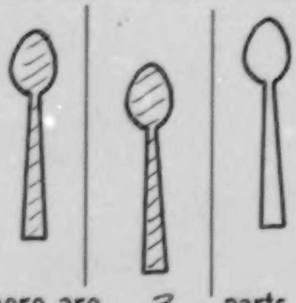
There are 2 parts.

Color 1 part.



There are 2 parts.

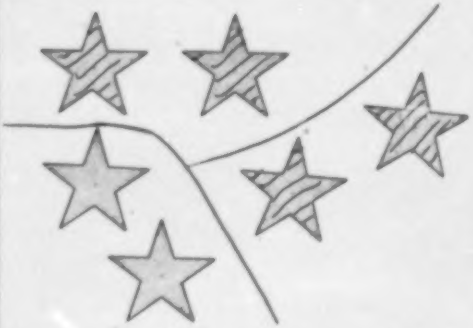
Color 2 parts.



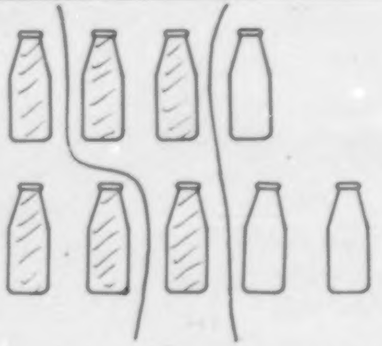
There are 3 parts.

Color 2 parts.

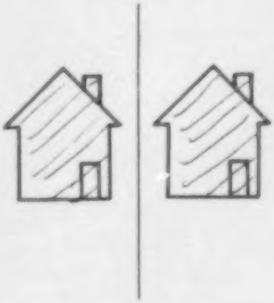
## Parts of Sets



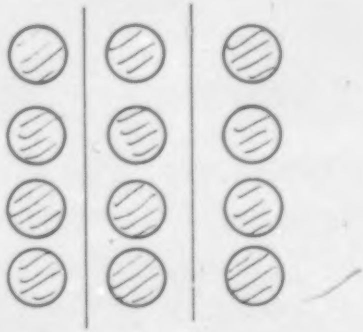
There are 3 parts. Color 2 parts.



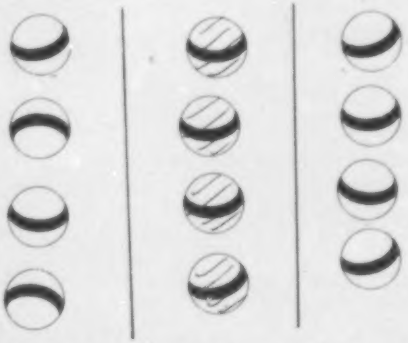
There are 3 parts. Color 2 parts.



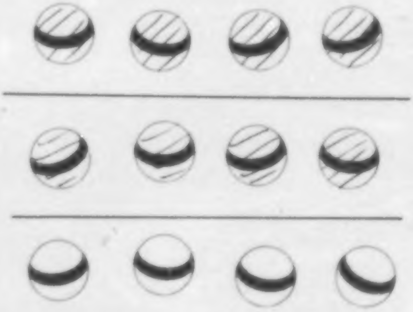
There are 2 parts. Color 2 parts.



There are 3 parts. Color 3 parts.



There are 3 parts. Color 1 part.



There are 3 parts. Color 2 parts.

## 9-4. ONE-HALF

OBJECTIVE: To introduce the idea of one-half, and the symbol for one-half, also, the idea of two halves, and the symbol for two halves.

VOCABULARY: One-half, two halves.

MATERIALS: Materials for flannel board--sets of objects, rectangular and circular regions.

Numeral cards $\boxed{\frac{1}{2}}$ $\boxed{\frac{2}{2}}$ to be used on the flannel board. Sets of objects for children to use on desk and the numeral cards $\boxed{\frac{1}{2}}$ and $\boxed{\frac{2}{2}}$.

### SUGGESTED PROCEDURE

### PRE-BOOK ACTIVITIES:

Place 6 flannel cut-outs (apples) on the flannel board in no particular arrangement. Ask children how many apples are on the flannel board. Then say--

Here is 1 set of apples.

Let's partition this set into 2 equivalent parts.

Who can show the two parts? (Let a child who thinks he can show the two equivalent subsets of apples, do so.)

Does anyone know what number describes this part of the starting set that we have on the flannel board? (Indicate one of the 2 subsets.)

Some child may be able to associate the number, one-half, with the part of the set. If they say, "Three," acknowledge that three describes the number of apples but we want a number to describe the part of one set.

If no one knows the number, tell the children that we have a new kind of number, a number whose name is one-half. Continue the discussion having children pretend that they are to give the two parts of the set of apples to two children, each child receiving one part.

Ask,

What number describes the part of the set of apples that Susan has? (One-half.)

What number describes the part of the set of apples that John has? (One-half.)

Then tell the children that the name of this number is written like this--as you exhibit the numeral card $\boxed{\frac{1}{2}}$. Have them look at the two cards on their desks. Ask which of the cards names the number, one-half. Point out that the numeral "$\frac{1}{2}$" is made by using the numerals for one and two. You may wish to ask if there are other numerals which are made by using 1 and 2. (12 and 21.) Emphasize that we write these names in a different way. We write "1", put a bar under it, and write "2" under the bar. (We do not expect children to write these numerals. They are just to recognize the numeral for one-half and later two-halves.)

Let us look at the set of apples again.
How many one-halves are shown? (2.)
The number that describes the whole set which has been separated into two equivalent parts is two-halves.
We write the name of this number like this--(exhibit the numeral card $\boxed{\frac{2}{2}}$ ).

Because many children have only the idea that one-half means a part of something, or less than all of a set, it is necessary to emphasize that finding one-half of a set required partitioning the set into 2 equivalent subsets.

Provide experiences for showing one-half and two halves of many different sets--4, 8, 10, 12. It is important that children learn to think of the set of objects as one set and that to find one-half of the set, they partition (or separate) the set into two equivalent subsets.

Take a sheet of paper. Have the children observe the sheet of paper. Place it on the chalkboard or some place where it can be kept in view. Then take another sheet of paper and show by placing it on top of the first sheet that the sheets are the same size. One fits exactly on the other.

Here is another sheet of paper the same size.
Let's separate this sheet into 2 parts. (Fold the paper and cut it.)
Are these parts the same size? (Yes.)
How can we be certain? (Put one piece on top of the other. See if they fit exactly.)
If the number 1 describes this sheet of paper (indicate the sheet

on the chalkboard), *what number describes this part of a sheet of paper* (hold up one of the two pieces)? (One-half.)

Ask a child to identify the numeral for this number by showing the numeral card which has its name. Then show both pieces of the sheet that has been cut.

*What number describes the paper that I now have in my hand?* (If children answer 1, accept this answer. However, also tell them that they can name the number, two-halves.)

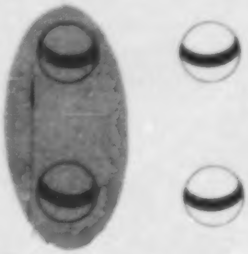
Ask children to show the numeral card which names two-halves. Continue with other regions where first the unit is identified and then it is separated into two parts of the same size. Identify the part that can be described by one-half and by two-halves.

Again, it is important that children learn that the number one-half describes that part of a set of 1 which has been separated into two equivalent parts and that two-halves describes two such parts.

*USING THE PUPIL'S BOOK*, pages 252-254:

Children are to ring the word "Yes" if the shaded part of the set can be described by one-half or by two-halves. They then ring the name of the number. If neither one-half nor two-halves describes the shaded part, then they ring the word, "No."

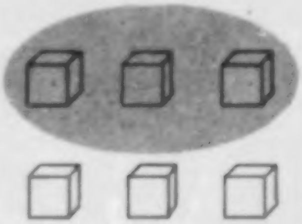
# One Half



[✓ $\frac{1}{2}$]

$\frac{2}{2}$

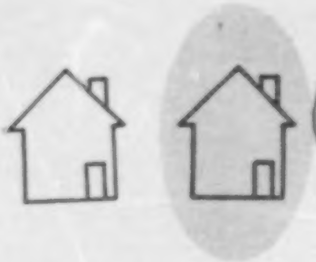
[✓ YES] NO



[✓ $\frac{1}{2}$]

$\frac{2}{2}$

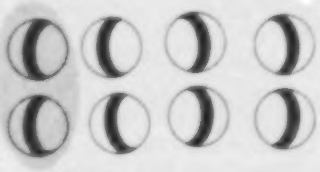
[✓ YES] NO



[✓ $\frac{1}{2}$]

$\frac{2}{2}$

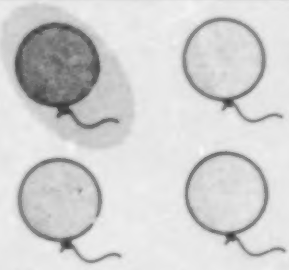
[✓ YES] NO



$\frac{1}{2}$

$\frac{2}{2}$

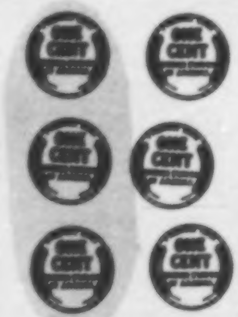
YES [✓ NO]



$\frac{1}{2}$

$\frac{2}{2}$

YES [✓ NO]

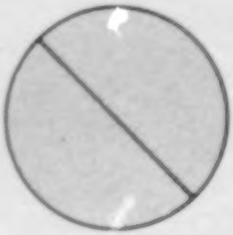


[✓ $\frac{1}{2}$]

$\frac{2}{2}$

[✓ YES] NO

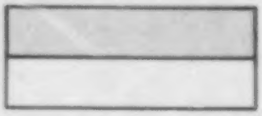
## One Half



$\frac{1}{2}$

[✓ $\frac{2}{2}$]

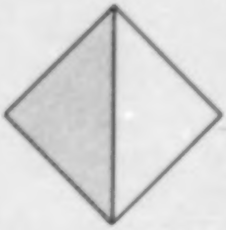
[✓ YES] NO



[✓ $\frac{1}{2}$]

$\frac{2}{2}$

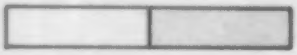
[✓ YES] NO



[✓ $\frac{1}{2}$]

$\frac{2}{2}$

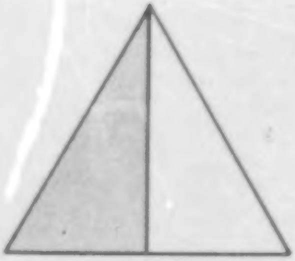
[✓ YES] NO



[✓ $\frac{1}{2}$]

$\frac{2}{2}$

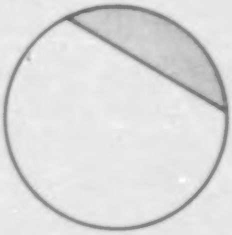
[✓ YES] NO



[✓ $\frac{1}{2}$]

$\frac{2}{2}$

[✓ YES] NO

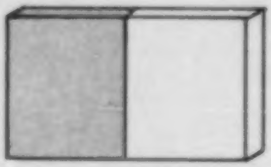


$\frac{1}{2}$

$\frac{2}{2}$

YES [✓ NO]

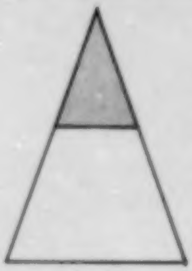
## One Half



[✓ $\frac{1}{2}$]

$\frac{2}{2}$

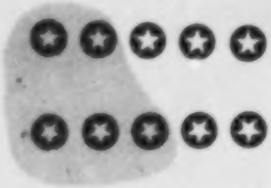
[✓ YES] NO



$\frac{1}{2}$

$\frac{2}{2}$

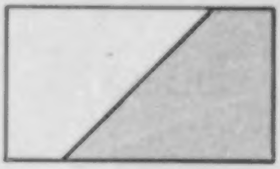
YES [✓ NO]



[✓ $\frac{1}{2}$]

$\frac{2}{2}$

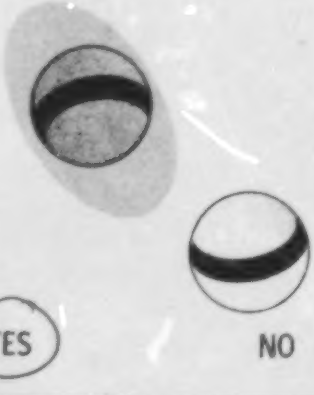
[✓ YES] NO



[✓ $\frac{1}{2}$]

$\frac{2}{2}$

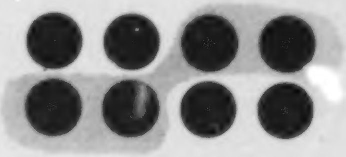
[✓ YES] NO



[✓ $\frac{1}{2}$]

$\frac{2}{2}$

[✓ YES] NO



[✓ $\frac{1}{2}$]

$\frac{2}{2}$

[✓ YES] NO

## 9-5. ONE-THIRD

**OBJECTIVE:** To introduce the idea of one third, and the symbol for one-third, also, two thirds and three thirds.

**VOCABULARY:** One-third, two-thirds, three-thirds.

**MATERIALS:** Sets of materials for the flannel board, rectangular and circular regions cut into three congruent parts. The numeral cards $\boxed{\frac{1}{3}}$ $\boxed{\frac{2}{3}}$ $\boxed{\frac{3}{3}}$ to be used on flannel board. A set of 15 counters (disks, buttons, etc.) for each child. Three numeral cards for each child $\boxed{\frac{1}{3}}$ $\boxed{\frac{2}{3}}$ $\boxed{\frac{3}{3}}$, also numeral cards used previously.

**SUGGESTED PROCEDURE**

**PRE-BOOK ACTIVITIES:**

Make a display of 12 disks on the flannel board. Review the idea of one-half by asking the children to imagine that the disks are cookies and that two children are to share the cookies. Each child is to have just as many cookies as the other. Ask two children to come to the board and decide how they might share these. If a suggestion is needed, ask each to take a cookie, and then each to take another cookie, and so on until all cookies are gone.

Then ask the first child to place his share of the cookies in a row. Ask the second child to place his share in a row below that of the first child. Then ask other children to name the number that describes the part of the set that each child has. They can indicate this by using their numeral cards. Together the children have how many halves? Again, the children respond by using their numeral cards. Again, arrange the 12 disks on the flannel board. Ask how three children can share the twelve cookies so that each child will have the same number of cookies. Ask three children to come to the board and decide how to do this. They may each take a cookie, then each take another cookie, and continue this process until all the cookies are gone. Ask the first child to place his share of the cookies in a row, the next child to place his in a row below that of the first child's cookies, and so on.

Ask if anyone knows what number describes the part of the set of cookies each child has. If no one says, "One-third," recall that we started with 1 set. The number which describes the part one child (name the child) has is one-third.

Have the numeral cards to be used displayed. Which of these do you think names one-third? Identify the name for one-third. ($\frac{1}{3}$)

Then talk about the cookies that two children have as being two-thirds of the one set of cookies. Identify the name for this number. ($\frac{2}{3}$) Children have three-thirds of the set of cookies or all of the cookies. Identify the name for this number. ($\frac{3}{3}$)

Repeat with other sets of objects until the idea that one-third is the number which describes one of three equivalent parts of the set, that two-thirds describes two such parts, and that three-thirds describes the three parts or the entire set. Children tell what number describes the part indicated by using their numeral cards.

Have children work individually with sets of objects, showing what part of a set of 6 can be described by one-third, two-thirds, three-thirds. They can indicate the part of the set by putting a piece of string around that part. Give them instructions orally and by indicating the parts using the numeral cards. In a similar way, they work with sets of 9, 3, 12, and 15 objects.

Follow procedures similar to those used for developing the understanding of one-half, using sheets of paper. Then continue with other circular and rectangular regions. First, they should observe the starting unit. Then another unit the same size is separated into three congruent parts.

It is important in this lesson that children learn that the number, one-third, describes 1 of the 3 equivalent or congruent parts into which the starting set of objects or region has been partitioned. Also, they learn that three-thirds describes the entire set or region.

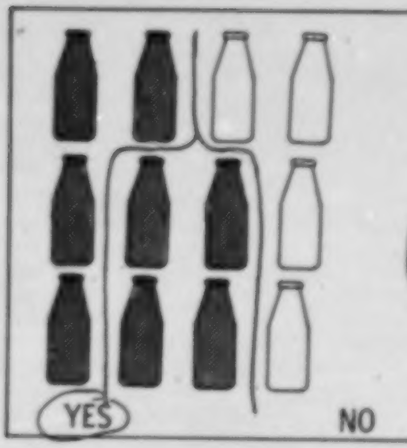
USING THE PUPIL'S BOOK, pages 255-256:

Children are to ring the word "Yes" if the shaded part of the set or region can be described by $\frac{1}{3}$, $\frac{2}{3}$, or $\frac{3}{3}$. Then they ring the numeral that names the number. If the shaded region cannot be described by these numbers, then they ring the word, "No."

Pages 257-258:

Children ring the correct numeral. If neither names the number which describes the shaded region, they ring "No."

## One-Third

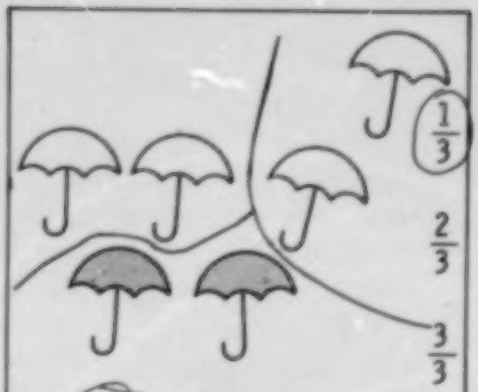


$\frac{1}{3}$

[✓ $\frac{2}{3}$]

$\frac{3}{3}$

[✓ YES] NO

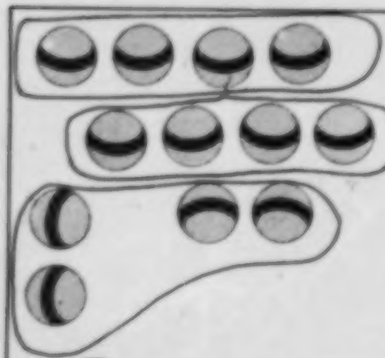


[✓ $\frac{1}{3}$]

$\frac{2}{3}$

$\frac{3}{3}$

[✓ YES] NO

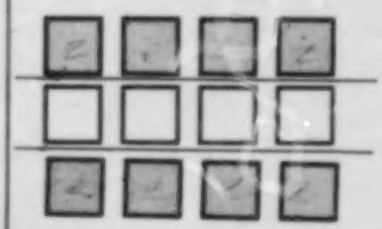


$\frac{1}{3}$

$\frac{2}{3}$

[✓ $\frac{3}{3}$]

[✓ YES] NO

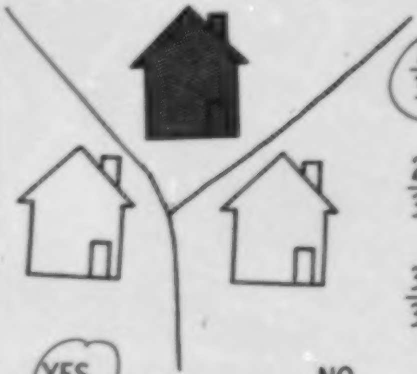


$\frac{1}{3}$

[✓ $\frac{2}{3}$]

$\frac{3}{3}$

[✓ YES] NO

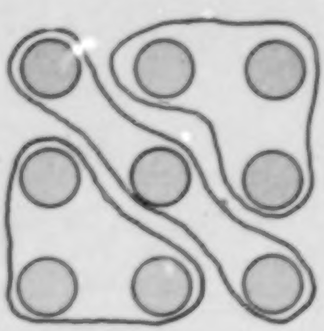


[✓ $\frac{1}{3}$]

$\frac{2}{3}$

$\frac{3}{3}$

[✓ YES] NO



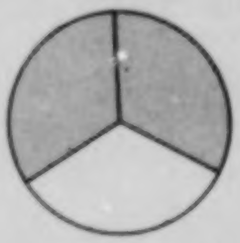
$\frac{1}{3}$

$\frac{2}{3}$

[✓ $\frac{3}{3}$]

[✓ YES] NO

## One-Third

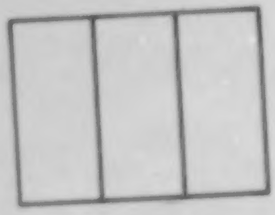


$\frac{1}{3}$

[✓ $\frac{2}{3}$]

$\frac{3}{3}$

[✓ YES] NO

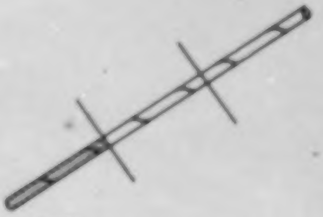


$\frac{1}{3}$

$\frac{2}{3}$

$\frac{3}{3}$

YES [✓ NO]

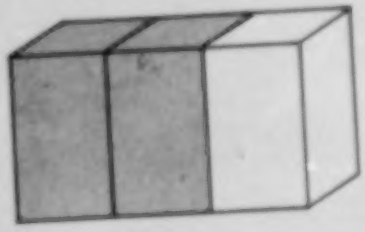


[✓ $\frac{1}{3}$]

$\frac{2}{3}$

$\frac{3}{3}$

[✓ YES] NO

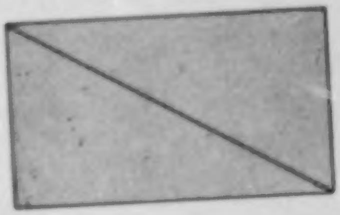


$\frac{1}{3}$

[✓ $\frac{2}{3}$]

$\frac{3}{3}$

[✓ YES] NO

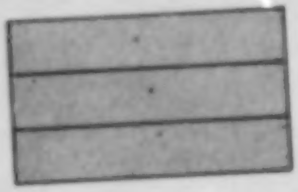


$\frac{1}{3}$

$\frac{2}{3}$

$\frac{3}{3}$

YES [✓ NO]



$\frac{1}{3}$

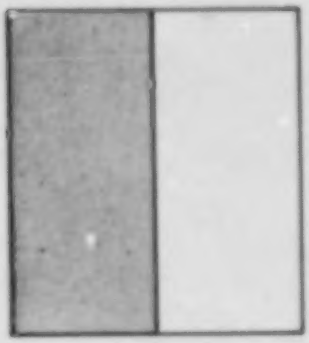
$\frac{2}{3}$

[✓ $\frac{3}{3}$]

[✓ YES] NO

## Regions

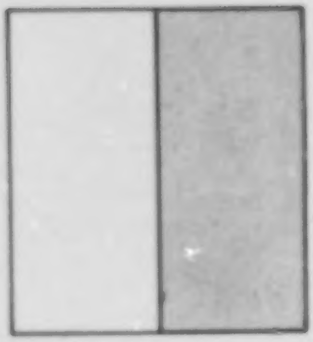
Ring the correct numeral



[✓ $\frac{1}{2}$]

$\frac{1}{3}$

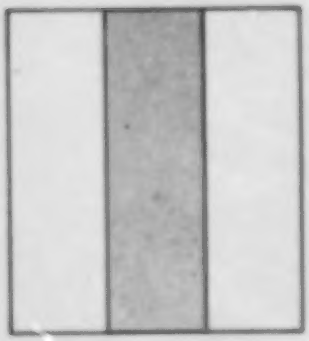
NO



$\frac{1}{3}$

[✓ $\frac{1}{2}$]

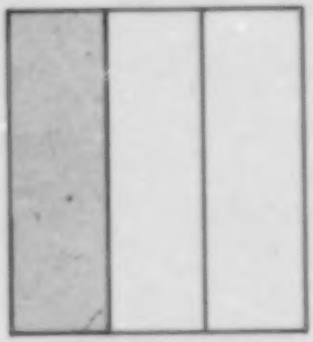
NO



[✓ $\frac{1}{3}$]

$\frac{1}{2}$

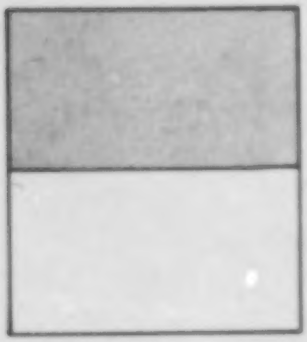
NO



[✓ $\frac{1}{3}$]

$\frac{1}{2}$

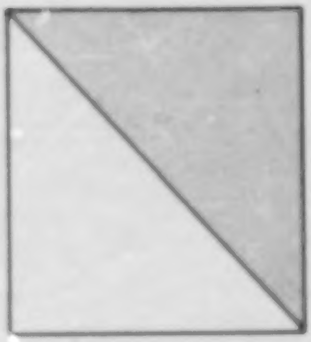
NO



[✓ $\frac{1}{2}$]

$\frac{1}{3}$

NO

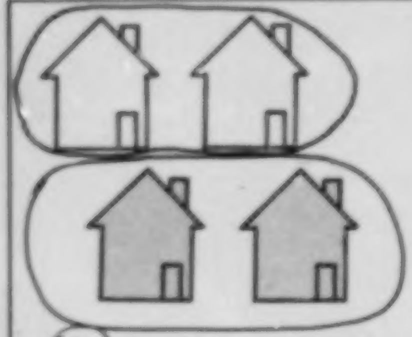


$\frac{1}{3}$

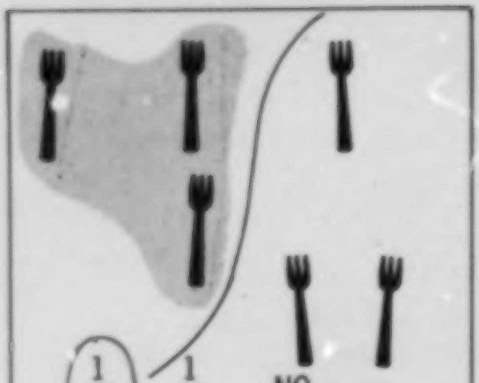
[✓ $\frac{1}{2}$]

NO

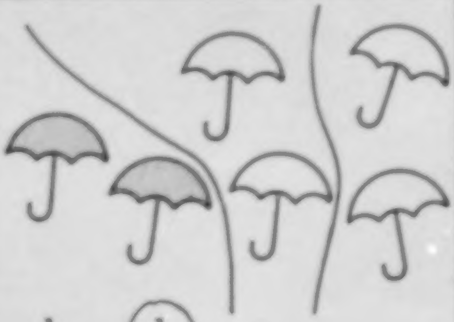
**Sets of Objects**
**Ring the correct numeral.**



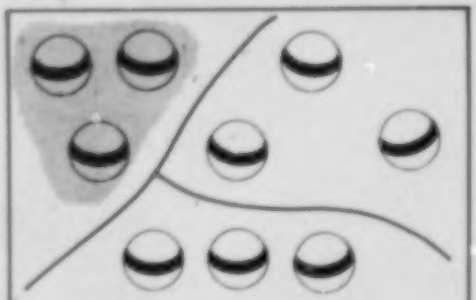
[✓ $\frac{1}{2}$] $\frac{1}{3}$ NO



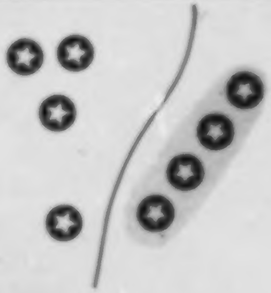
[✓ $\frac{1}{2}$] $\frac{1}{3}$ NO



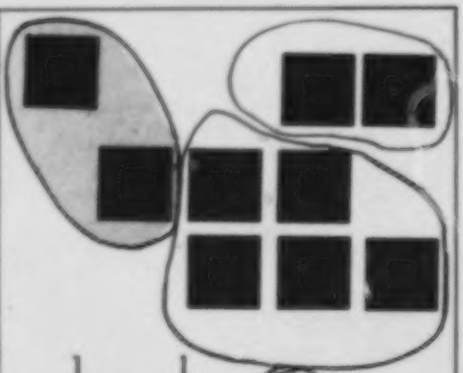
$\frac{1}{2}$ [✓ $\frac{1}{3}$] NO



$\frac{1}{2}$ [✓ $\frac{1}{3}$] NO



[✓ $\frac{1}{2}$] $\frac{1}{3}$ NO



$\frac{1}{2}$ $\frac{1}{3}$ [✓ NO]

## 9-6. USING RATIONAL NUMBERS

**OBJECTIVE:** To find the number of objects in one-half and one-third of a given set.

**VOCABULARY:** (No new words.)

**MATERIALS:** A set of 12 objects (apples) to be used on the flannel board; a set of 12 counters (disks, buttons, etc.) for each child.

### SUGGESTED PROCEDURE

**PRE-BOOK ACTIVITIES:**

Place a set of 8 objects (apples) on the flannel board.

Suppose that we put these apples on two plates.
We will let these pieces represent the plates. (Place two flannel strips on the flannel board.)
We will put just as many apples on one plate as on the other.
Who can separate the set of apples into two equivalent subsets?
(Ask child who thinks he can do this to do so.)
What part of the set of apples is on this plate (indicate plate)? ($\frac{1}{2}$)
What part of the set of apples is on this plate (indicate other plate)? ($\frac{1}{2}$)
How many apples are on this plate? (Again, indicate first plate.) (4.)
How many apples are on this plate? (Indicate other plate.) (4.)
Let's write sentences that describe what we have done.

Again ask for the number of apples that were placed on the flannel board. Then write:

There are___________apples.
How many apples are in $\frac{1}{2}$ of the set? (4.)
We write:

$$\frac{1}{2} \text{ of } 8 \text{ is } 4.$$

Continue with other sets of apples--6 apples, 4 apples, 2 apples, 10 apples. Each set is then partitioned into two equivalent parts. Each part is identified as one-half of the set of apples. Then ask for the number of apples in each part. When those actions and oral descriptions have been given, then write the sentence that can be associated with finding $\frac{1}{2}$ of the set of objects.

For example, $\frac{1}{2}$ of 6 is 3.
$\frac{1}{2}$ of 4 is 2.
$\frac{1}{2}$ of 2 is 1.
$\frac{1}{2}$ of 10 is 5.
$\frac{1}{2}$ of 1 is $\frac{1}{2}$.

Ask children to use sets of objects on their desks. Use sets of 4, 6, 8, 2, and 10. They first show one-half of the set. Then they tell how many objects are in that part of the set. You write on the chalkboard the sentence that describes finding $\frac{1}{2}$ of each set of objects.

Read the following word problems to the children. Let them use the objects on their desks to find the answers. Then you write the sentences that can be associated with the problem on the chalkboard. After they find the answer, write the numeral in the blank to complete the sentence.

1. 6 boys were playing ball. One-half of the boys went home. How many boys went home? (3.) What part of the group was still playing ball? (3.)
   ($\frac{1}{2}$ of 6 is _____.)
2. Mother had 8 sticks of gum. She gave $\frac{1}{2}$ of the gum to Mary. How many sticks of gum did mother give to Mary? (4.)
   ($\frac{1}{2}$ of 8 is _____.)
3. John had 4 cookies. He gave 2 cookies to Tom. What part of the set of cookies did Tom get? ($\frac{1}{2}$.) What part of the set of cookies did John still have? ($\frac{1}{2}$.)
   (_____ of 4 = 2.)
4. Father had 12 nails. He used $\frac{1}{2}$ of the nails to make a bird house. How many nails did he use? (6.)
   ($\frac{1}{2}$ of 12 is _____.)
   What part of the set of nails did he still have? ($\frac{1}{2}$.)

USING THE PUPIL'S BOOK, pages 259-260:

Children first color the objects to show the two equivalent parts of the set. Then they complete the sentence.

Use similar experiences for using the number one-third, first starting with a set of 12 objects (apples) on the flannel board. Then put them on three plates, etc.

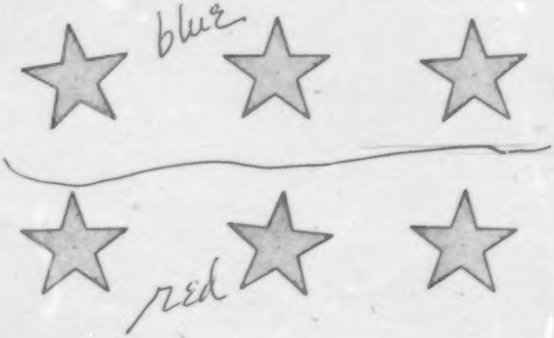
Continue the discussion with other sets of objects on the flannel board, as, sets of 6, 9, 3, and 1. Also have children use objects on their desks. Now they separate the set into 3 parts and identify $\frac{1}{3}$ of the set.

Pages 261-262:

Children first color the objects to show the three equivalent parts of the set. Then they complete the sentence.

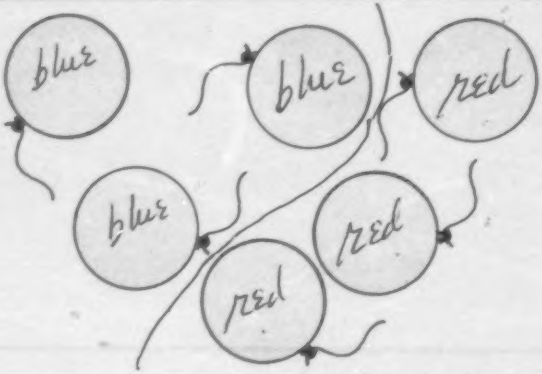
## One Half

Color $\frac{1}{2}$ of each set blue. Color the other half red. Fill the blanks.



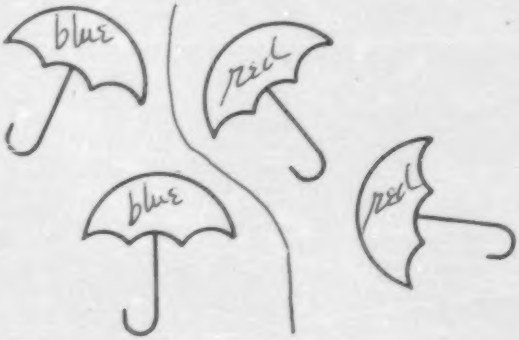
There are 6 stars.

$\frac{1}{2}$ of 6 is 3.


blue
blue
red
blue
red
red

There are 6 balloons.

$\frac{1}{2}$ of 6 is 3.

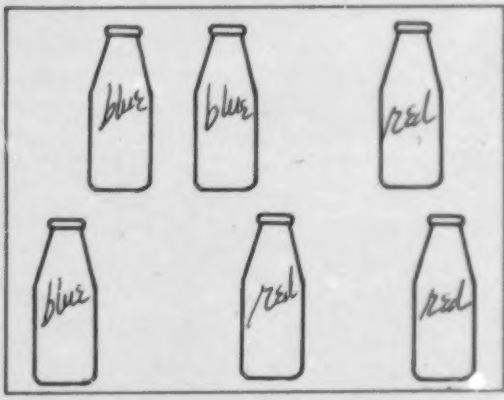

blue
red
blue
red

There are 4 umbrellas.

$\frac{1}{2}$ of 4 is 2

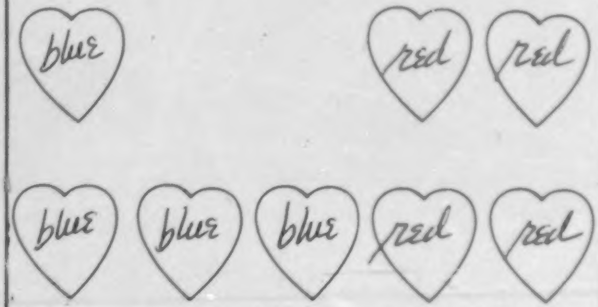
## One Half

Color $\frac{1}{2}$ of each set blue. Color the other half red. Fill in the blanks.


blue
blue
red
blue
red
red

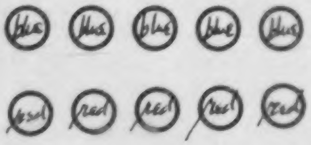
There are 6 bottles.

$\frac{1}{2}$ of 6 is 3.


blue
red
red
blue
blue
blue
red
red

There are 8 hearts.

$\frac{1}{2}$ of 8 is 4.

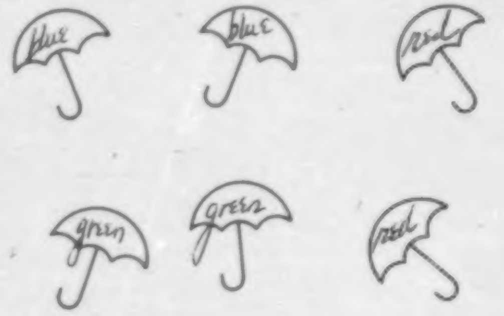

blue
blue
blue
blue
blue
red
red
red
red
red

There are 10 circles.

$\frac{1}{2}$ of 10 is 5.

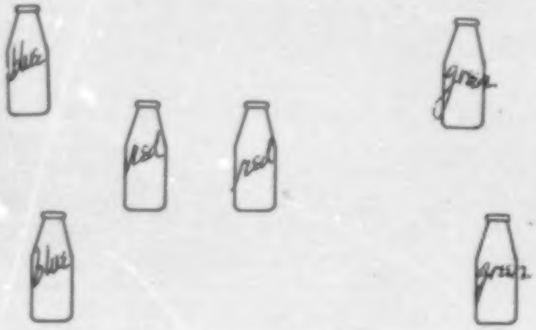
## One Third

Color $\frac{1}{3}$ of each set blue. Color $\frac{1}{3}$ of each set red. Color $\frac{1}{3}$ of each set green. Fill in the blanks.


blue
blue
red
green
green
red

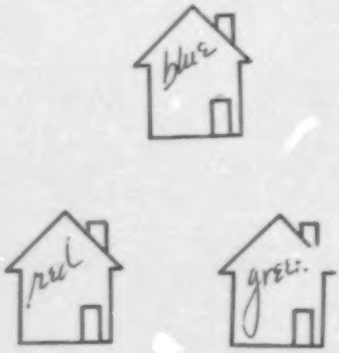
There are 6 umbrellas.

$\frac{1}{3}$ of 6 is 2.


blue
green
red
red
blue
green

There are 6 milk bottles.

$\frac{1}{3}$ of 6 is 2.

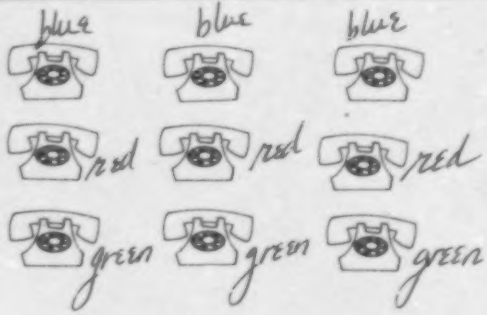

blue
red
green

There are 3 houses.

$\frac{1}{3}$ of 3 is 1.

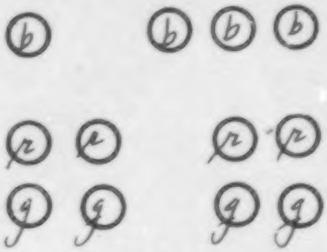
## One Third

Color $\frac{1}{3}$ of each set blue. Color $\frac{1}{3}$ of each set red. Color $\frac{1}{3}$ of each set green. Fill the blanks.


blue
blue
blue
red
red
red
green
green
green

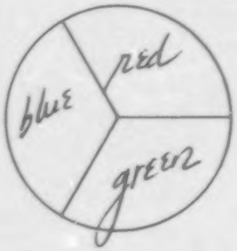
There are 9 telephones.

$\frac{1}{3}$ of 9 is 3.


b
b
b
b
r
r
r
r
g
g
g
g

There are 12 circles.

$\frac{1}{3}$ of 12 is 4.


red
blue
green

There is 1 region.

$\frac{1}{3}$ of 1 is $\frac{1}{3}$.

# Chapter 10

# LINEAR MEASUREMENT

## BACKGROUND

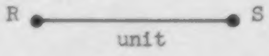
In this chapter, we discuss the measurement of line segment. Recall that a *line segment* is the set of points followed in passing along a straight path from a given point A to a given point B. Two line segments are *congruent* provided that they have the same size, so that one will fit exactly on the other.

Long before the child comes to school he has experience in comparisons of order: his father is taller than he is; his sister is younger than he is; the new house is bigger than the old house; this pail is heavier than that pail. He has also had experience with the notion of measure; he understands and makes such statements as, "My dad is 6 feet tall," "We get 3 quarts of milk a day," "It takes me 15 minutes to get to school." Here we wish to extend the child's knowledge and intuitive understanding of linear measure.

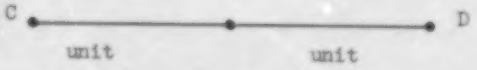
Our development parallels the historical one. The counting of separate objects (say, sheep) was a technique not applicable to measuring a region or curve (like a field or its boundary). Nevertheless, one can often make comparisons: this field is larger than that; this boundary is longer than that. Later, when one field bordered another, actual measurement became necessary. When a unit of measure (e.g., that part of a rope between two knots) was agreed upon, it was possible to designate a piece of property as having a length of "50 units of rope" and having a width of "30 units of rope". With the increase in travel and communication it became obvious that "50 units of rope" did not always represent the same length. Hence, standard units were adopted. For convenience in measuring, rules or scales marked in these *standard units* were introduced.

## MEASURE, LENGTH, UNITS

In measuring line segments, we first select a particular line segment, say $\overline{RS}$, to serve as a *unit*.



The length of $\overline{RS}$ itself is then 1 unit. To measure any given line segment $\overline{CD}$, we lay off the unit $\overline{RS}$ along it.



If the unit can be laid off exactly twice, as in the picture, we say that the measure of $\overline{CD}$ is 2, and that the length of $\overline{CD}$ is 2 units. If the unit could be laid off exactly three times, we would say that the measure of $\overline{CD}$ is 3, and that the length of $\overline{CD}$ is 3 units. The measure of a line segment is a number: the number of times the unit can be laid off on a line segment. When naming a length, we use both the measure and the unit.

## LENGTH TO THE NEAREST UNIT

More often than not, the unit will not fit exactly some whole number of times. There will be a part of a unit left over. In the picture below, the unit can be laid off along the segment $\overline{AB}$ 3 times, with a part of a unit left over, but it does not fit 4 times.

A
B
unit
unit
unit

The length of $\overline{AB}$ is then greater than 3 units but less than 4 units. Moreover, in our examples the length of $\overline{AB}$ is visibly nearer to 3 units than to 4 units. In this case, we say that the length of $\overline{AB}$ to the nearest unit is 3 units. This approximation is the best we can give without introducing fractional parts of a unit or shifting to a smaller unit. In this chapter, we will not introduce the phrase, to the nearest unit, but will note that the length of $\overline{AB}$ above is between 3 and 4 units.

## STANDARD UNITS AND SYSTEMS OF MEASURES

The acceptance of a standard unit for purposes of communication is soon followed by an appreciation of the convenience of having a variety of standard units. An inch is a suitable standard unit for measuring the edge of a sheet of paper, but hardly satisfactory for finding the length of the school corridor. While a yard is a satisfactory standard for measuring the school corridor, it would not be a sensible unit for finding the distance between Chicago and Philadelphia.

## 10-1. LINE SEGMENT, STRAIGHTEDGE

OBJECTIVE: To introduce the concept of line segment and the use of the straightedge.

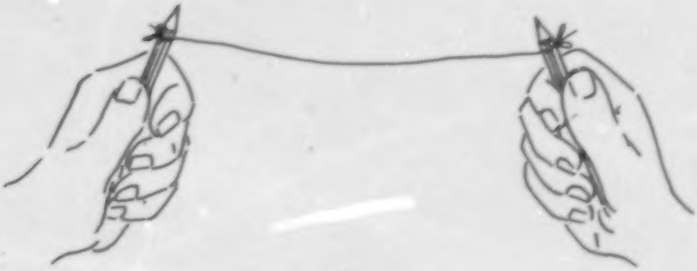
VOCABULARY: Straightedge, line segment.

MATERIALS: Jump rope, yarn, string, thread; various models of line segments; unmarked strips of cardboard (at least 10 inches in length).

### SUGGESTED PROCEDURE

### PRE-BOOK ACTIVITIES:

Show a loosely held string between two pencils. Pull the string tightly to demonstrate the idea of a straight path.

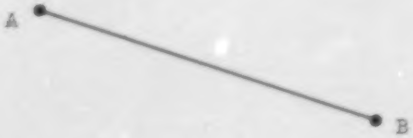


Ask the children to identify objects that display straight paths: the edge of a desk, a sheet of paper, etc.

Explain that these are all examples of line segments; a straightedge from one point to another. Call attention to the physical things that suggest the endpoints. For example, if the edge of a block is mentioned as a line segment, then the corners of the block represent the endpoints.

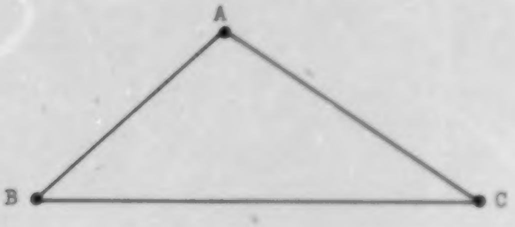
On the chalkboard show two points. Draw a line segment between them. Use an unmarked cardboard straightedge, since one will be used later by the children.

Explain that it is often helpful to give names to the endpoints. Label them A and B as shown.



Explain further that the names of the endpoints may be used to name the line segment either as $\overline{AB}$ or as $\overline{BA}$. Illustrate and name several other line segments.

Uncover on the chalkboard, a picture of a triangle. Ask the children if there is a way in which they can use line segments and letters to describe the triangle. Then label the triangle.



Help the children to visualize that this triangle can be described as being made up of line segment $\overline{AB}$, line segment $\overline{BC}$, and line segment $\overline{CA}$.

Show two more points on the board. Demonstrate a technique for using straightedge and chalk. Show that if a piece of chalk is placed on one point and the straightedge lined up slightly below the other point, then the line segment drawn will include both points. Also, discuss the importance of holding the straightedge at the center rather than at an end.

Distribute a cardboard strip to each child. Have each child put 2 dots on a sheet of paper and use his cardboard strip to draw a line segment with these dots as endpoints. Ask them to draw other line segments between other pairs of endpoints.

USING THE PUPIL'S BOOK, pages 263-264:

IDEAS: A line segment connects two points. A straightedge can be used to draw a line segment.

Page 263:

Give oral directions to draw line segments $\overline{AC}$, $\overline{EC}$, and $\overline{BD}$. Tell the children to place their pencils on point A, line up the straightedge with point C, hold it in the center, then draw $\overline{AC}$. Do the same for the second exercise on this page.

Page 264:

Read instructions and give help where needed. Some children may not think to count $\overline{AD}$ and $\overline{BC}$ as line segments. In discussion, help them see that the two shorter line segments are part of the longer line segment. Note also that two small triangular regions such as those bounded by $\triangle ABE$ and $\triangle ACE$ are part of the larger triangular region bounded by $\triangle ABC$.

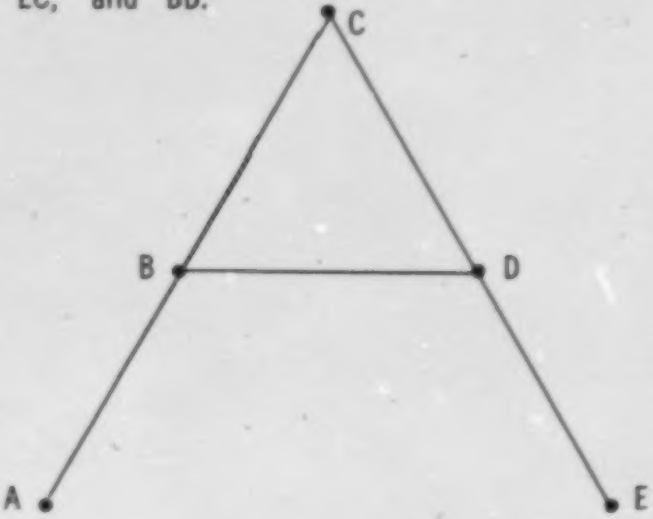
Pages 265-266:

Read instructions for both pages, then let children work independently. When page 266 is completed, ask the children to compare the two examples (what happens when point D is inside, outside, the triangle). They should learn that $\overline{AB}$ and $\overline{AC}$ mean line segment AB and line segment AC.

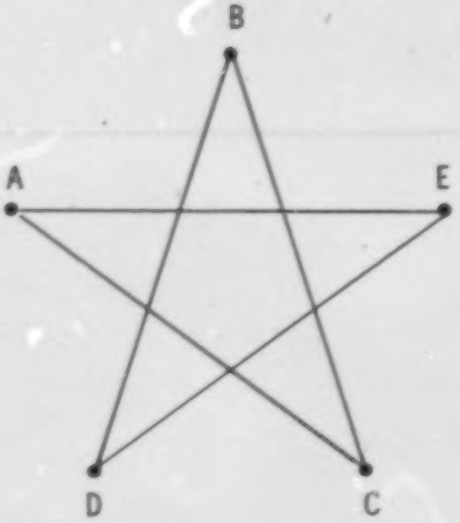
Each page presents one of the ideas of this section for visual comparison. Read the instructions with the children. Make sure that they agree that the marking of the first example on each page is correct.

## Line Segments

Draw $\overline{AC}$, $\overline{EC}$, and $\overline{BD}$.



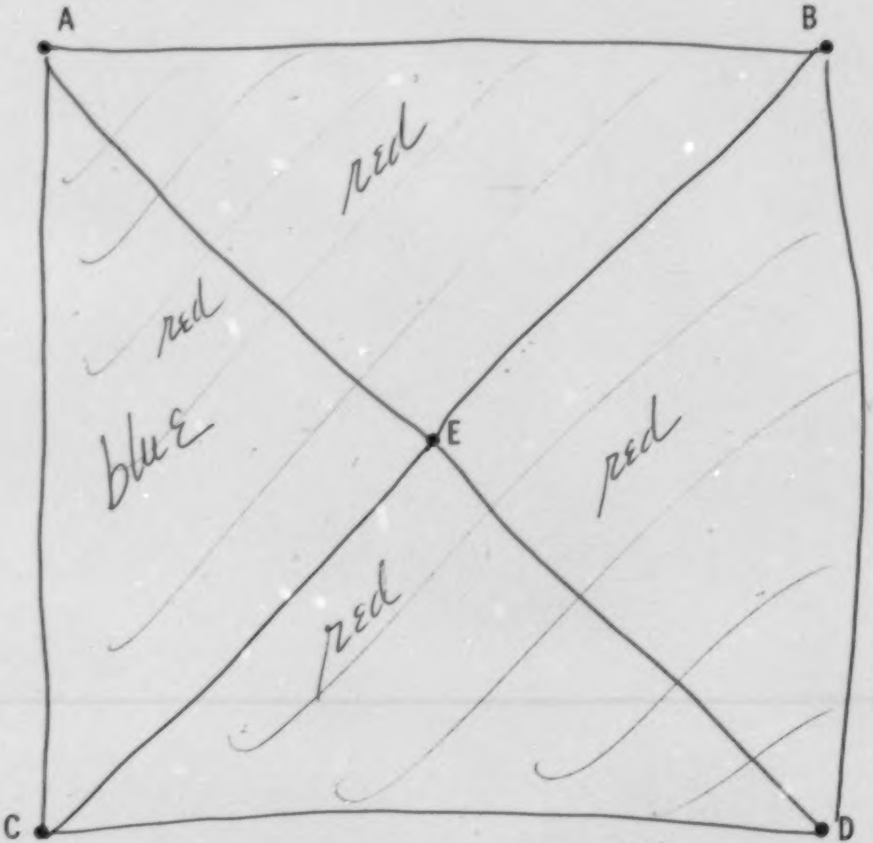
Draw $\overline{AE}$, $\overline{AC}$, and $\overline{DE}$



## Line Segments

Draw $\overline{AB}$, $\overline{BD}$, $\overline{DC}$ and $\overline{CA}$.

Connect point E with the other points.


A
B
red
red
blue
E
red
red
C
D

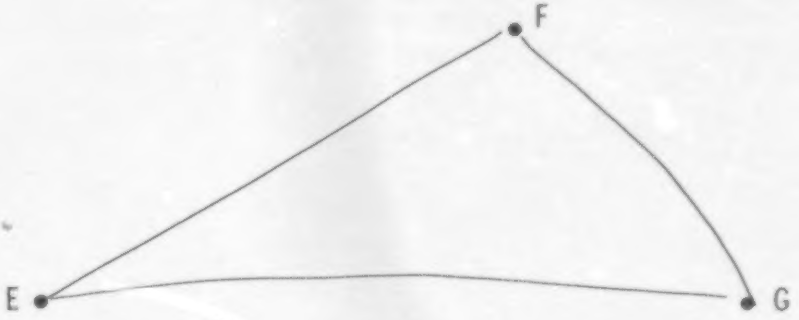
How many line segments can you count? 10

Color a square region red.

Color one triangular region blue.

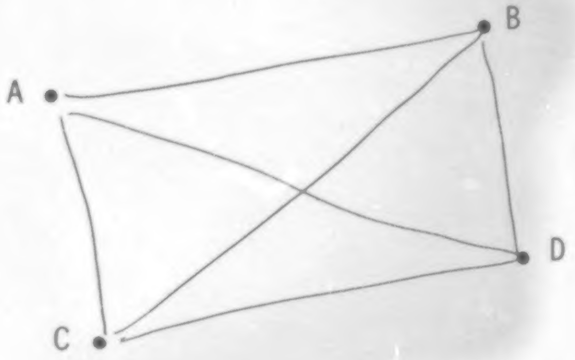
Line Segments

Connect each point by a line segment to each of the other points.



Do any line segments cross?

Mark Yes or No. Yes [✓ No]

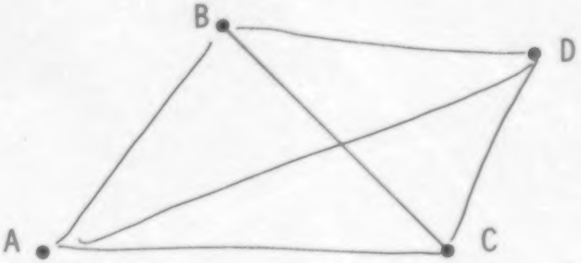


How many line segments cross? YES

## Line Segments

Draw $\overline{AB}$ and $\overline{AC}$.

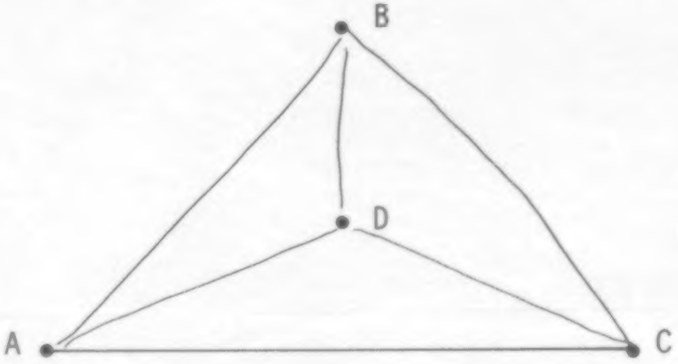
Now connect point D with the other points.



How many line segments cross? 2

Draw $\overline{BA}$ and $\overline{BC}$.

Now connect point D with the other points.



Do any line segments cross? Yes [✓ No]

## 10-2. COMPARING LINE SEGMENTS

OBJECTIVE: To introduce the ideas of longer than, longest, shorter than, shortest, same length as.

To compare line segments by using an intermediate model.

VOCABULARY: Compare, longer than, longest, shorter than, shortest, same length as.

MATERIALS: One long paint brush and one sho[illegible] paint brush for each child, several tagboard and chipboard sheets of varied lengths, flannel board, three strips of cloth of different lengths, individual pieces of string, each 8 inches long, and as needed, pencils, pipe cleaners, pick-up sticks, book, straws.

### SUGGESTED PROCEDURE

### PRE-BOOK ACTIVITIES:

### COMPARING LENGTHS OF OBJECTS

Give each child one short paint brush and one long paint brush. There should be a distinct difference in length between the brushes. If brushes are not available in quantity, use straws.

Ask the children to put the brushes on end on their desks. Find out how the brushes are alike. (Both are brushes, wood, etc.) Find out how they are different. (This brush is longer than that paint brush.)

Suggest to a child that he observe the brushes of the child next to him. Ask him to find a brush the same length as one of his, and to display the two. Continue with another child finding a brush longer than (shorter than) his.

Select the children at one table for demonstration. Give a paint brush to one child. Ask him to compare the brush with the two he has. Seek the response that the new brush is the same length as one of his paint brushes, and longer than the other paint brush. Repeat with different children, alternating with a short and a long brush.

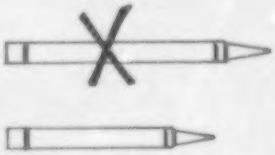
USING THE PUPIL'S BOOK, pages 267-269:

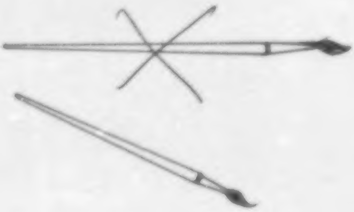
IDEAS: An object can be longer than, shorter than, or the same length as another object.

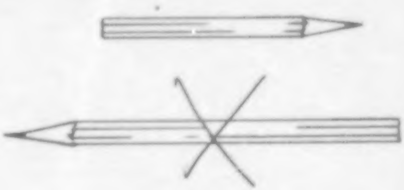
Each page presents one of the ideas of this section for visual comparison. Read the instructions with the children. Make sure that they agree that the marking of the first example on each page is correct.

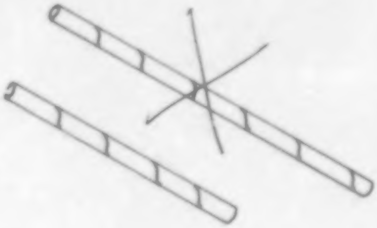
## Comparing Lengths

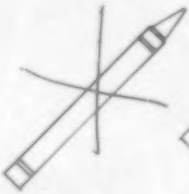
Mark the one that is longer than the other.

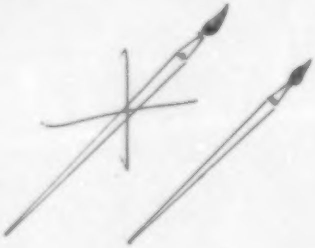


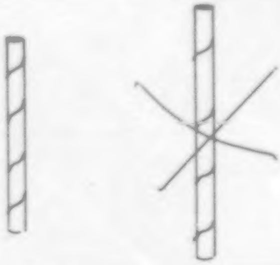


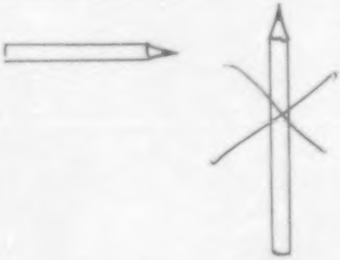






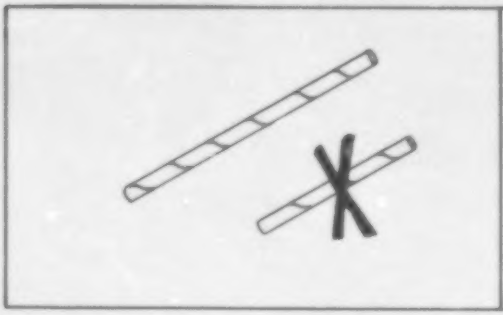


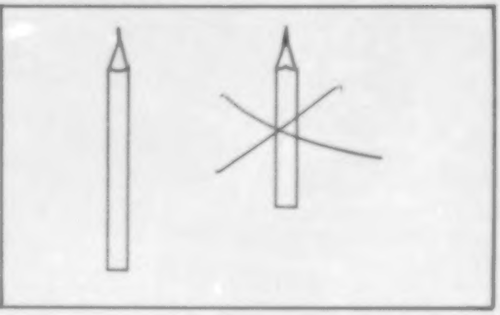


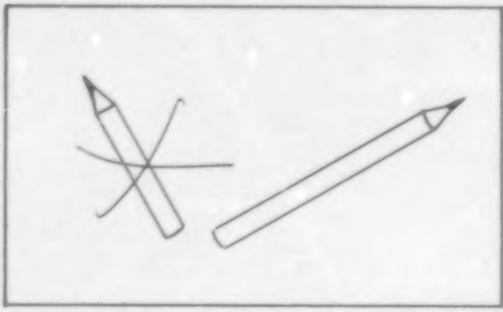


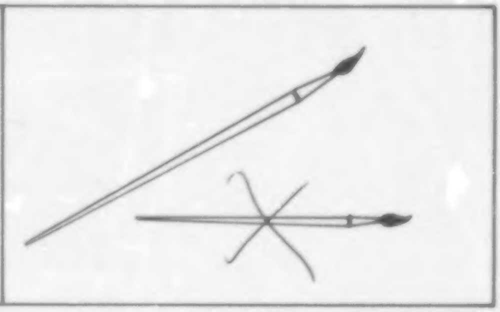
## Comparing Lengths

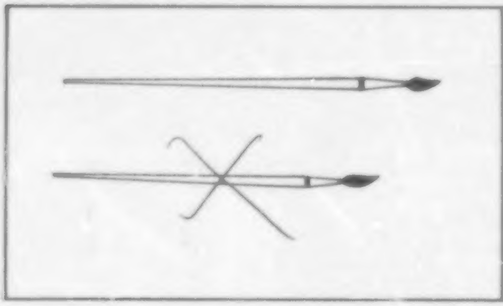
Mark the one that is shorter than the other.

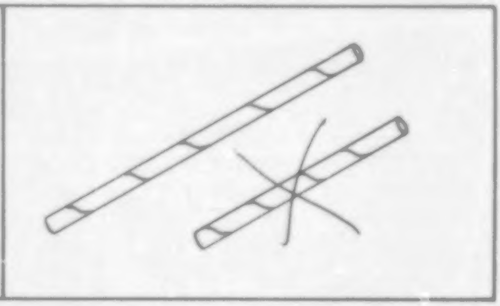


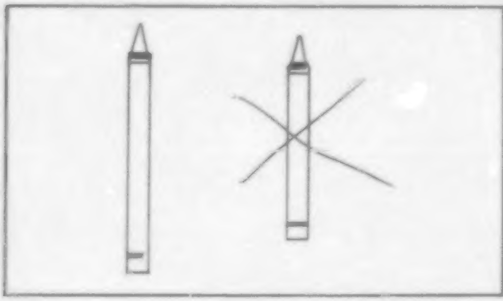


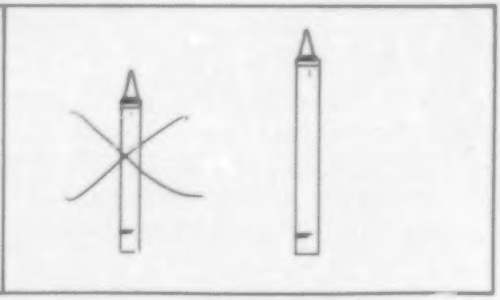






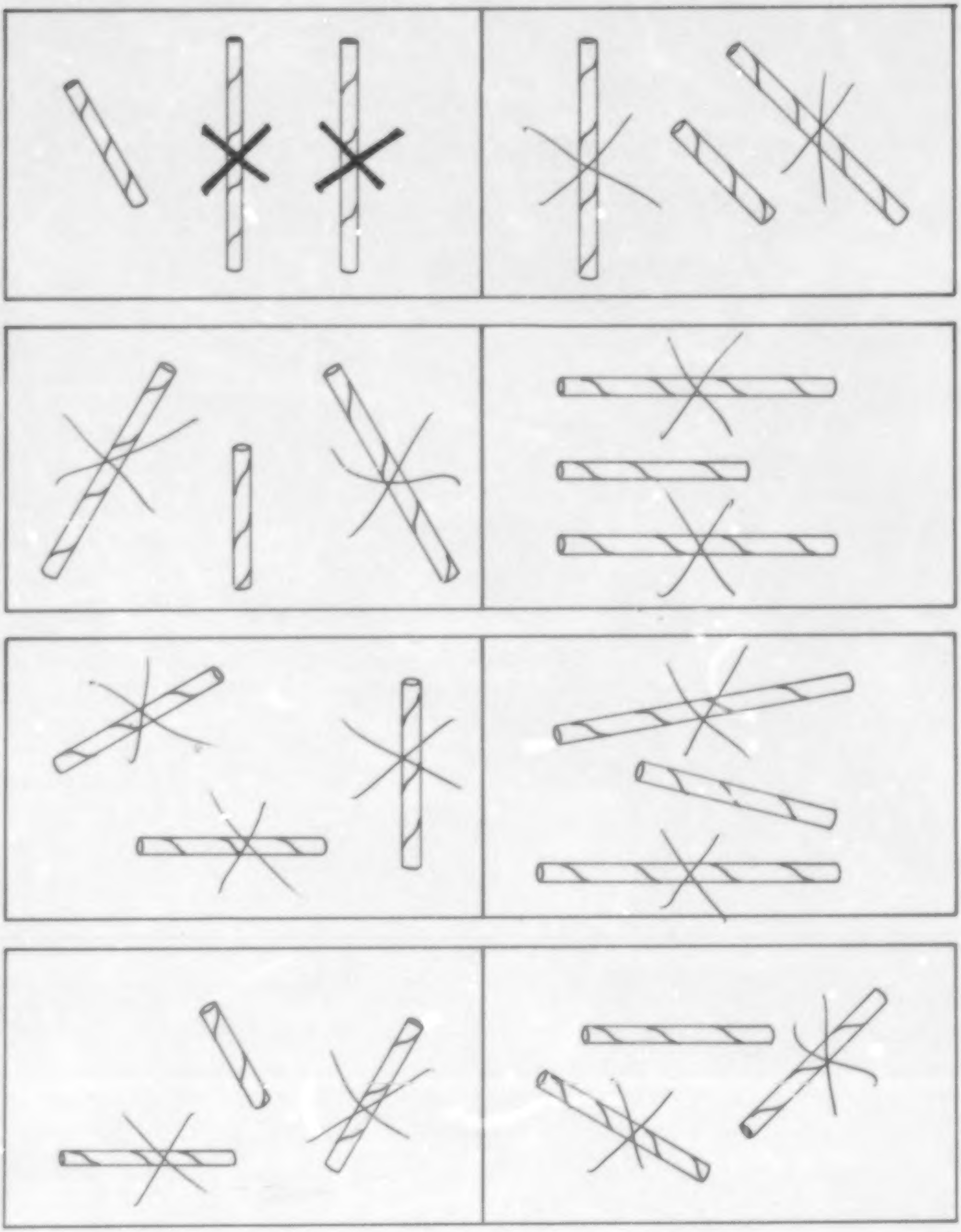






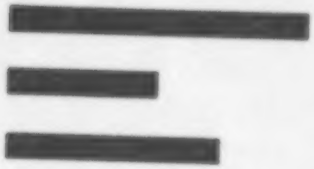
## Comparing Lengths

Mark the sticks that are the same length.



## COMPARING LINE SEGMENTS

Direct the children's attention to the flannel board where three strips of colored cloth of distinctly different lengths are displayed. These strips should be placed horizontally and have the same beginning position.



Discuss which strips are longer, then ask which is longest. (The one that is longer than any of the others). Repeat with shorter and shortest. Test for length by moving one edge against another.

In two parts of the room place two objects (fairly narrow) that are obviously not the same length. Compare them at a distance, then bring the objects together for comparison of their edges. Then place two objects that are the same length and repeat the comparison.

Introduce two narrow tagboard or chipboard sheets, one only slightly longer than the other. When the comparison is made, point out the advantage of being able to bring the objects together to check the lengths of their edges.

Call attention to two different edges of the flannel board (one edge should be shorter). Ask how these line segments could be compared. Accept any of the following ideas:

1. Holding one's hands at the ends of one line segment and using this to transfer to the other line segment. (The endpoints are marked by the hands. Point out the difficulties involved.)
2. Laying a piece of string beside one line segment, and then grasping it carefully at the endpoints of the line segment and carrying it over to the other line segment. (Clarify that the string represents the line segment, and the places where it is held the endpoints. The method is not very practical because the string may stretch if tension on it is increased.)

3. Using a long unmarked stick or piece of paper by placing one end of the stick or paper at one end of the line segment, marking a point on the object at the other end of the line segment, and then comparing the marked object with the other line segment. (Indicate that the edge of the stick from one end to the mark represents the line segment.)

Clarify that in each case above, in one way or another, a model has been made of one line segment. This model has been superimposed on the other segment for comparison.

Use string to show how the edges of the flannel board can be compared.

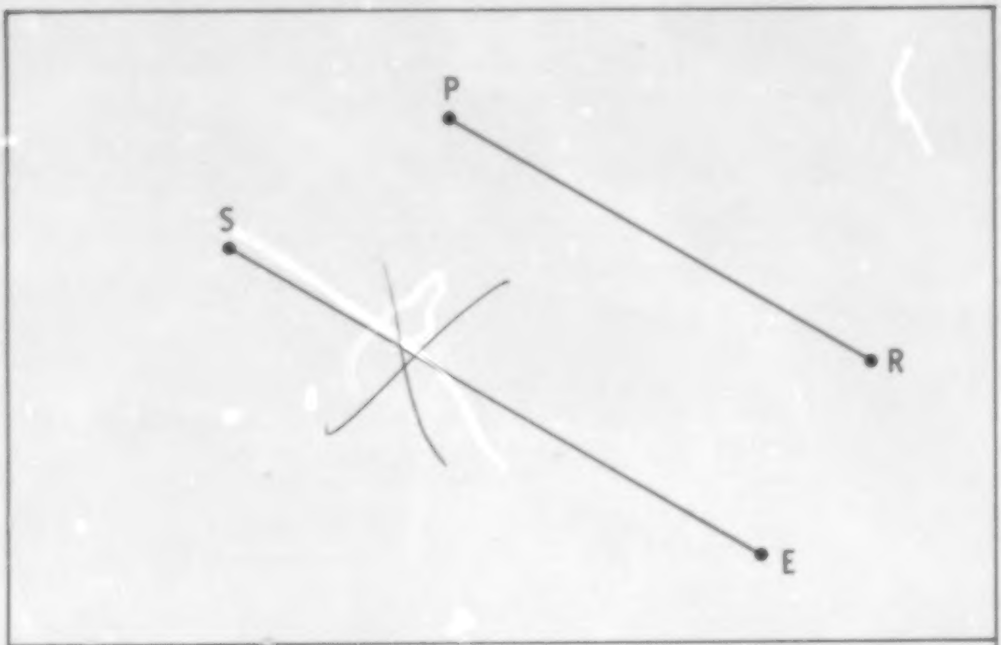
USING THE PUPIL'S BOOK, pages 270-273:

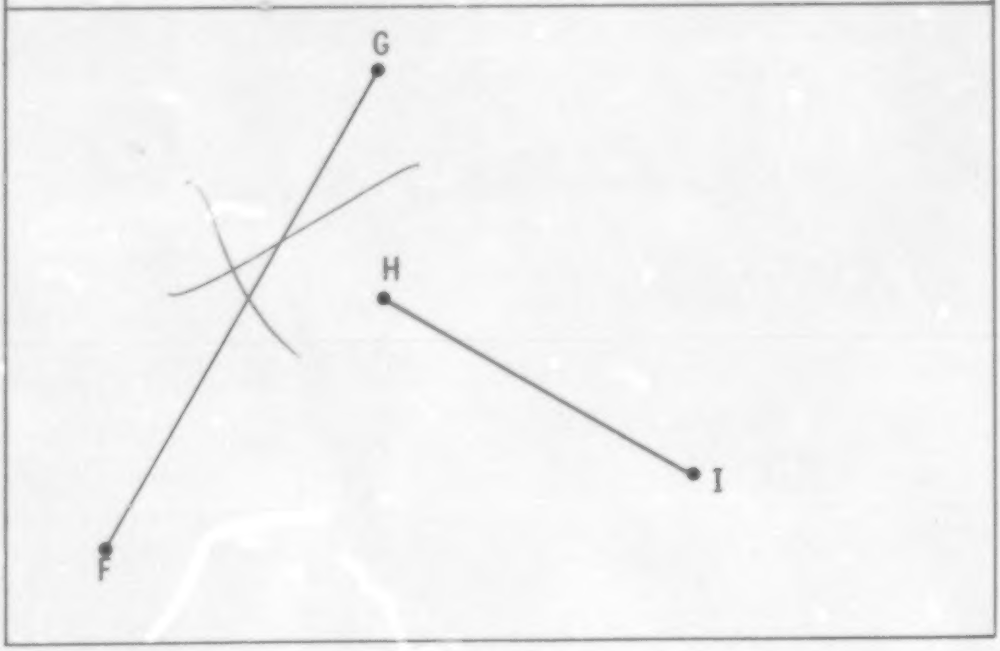
IDEAS: Two line segments can be compared by using a model of one and placing it on the other.

Pass out string to the class. Read the instructions and tell the children they are to use the string to compare the line segments in each set. Give no more instructions, but move around and ask leading questions to those who are obviously copying or are not able to get started.

## Comparing Line Segments

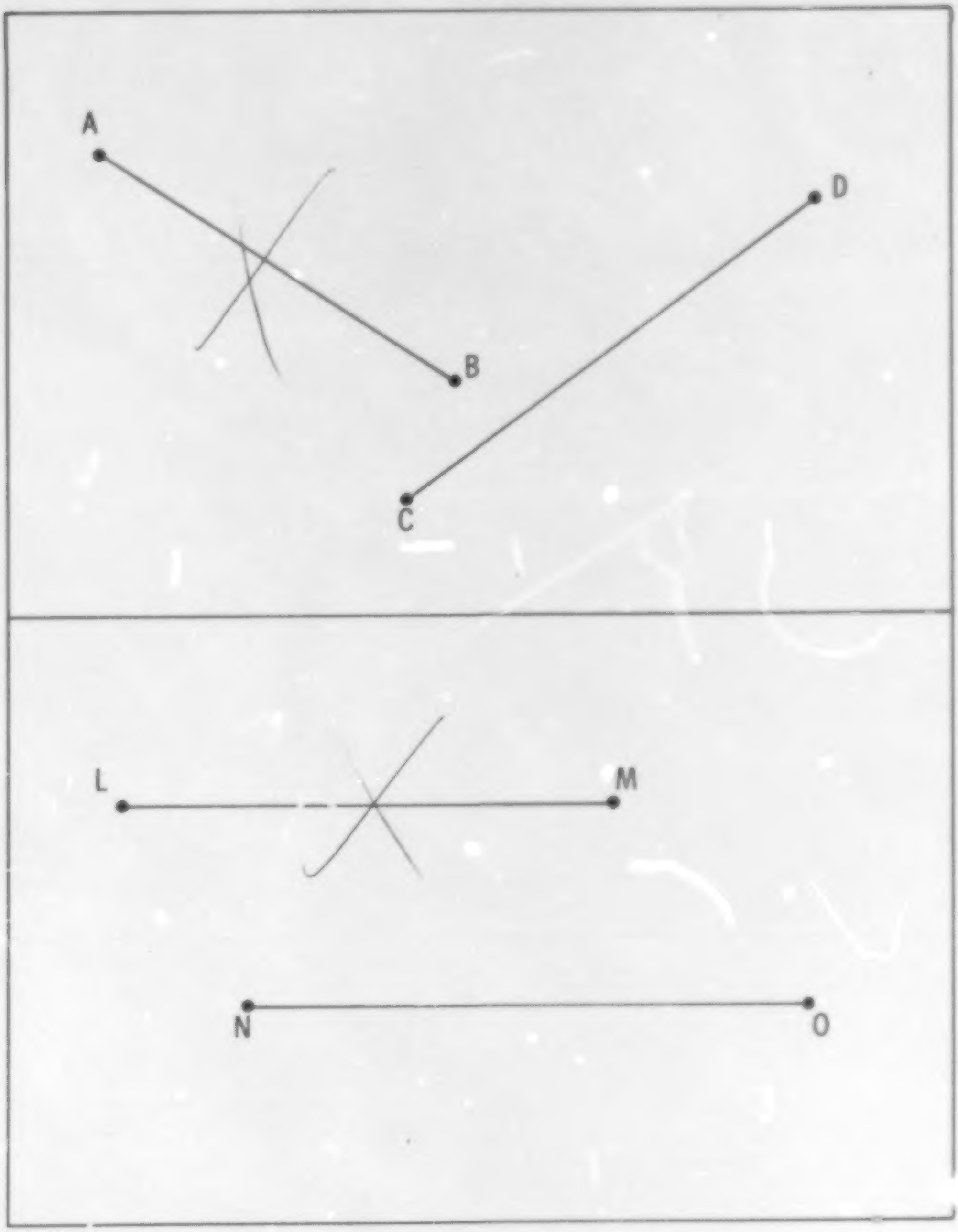
Mark the line segment that is longer than the other one.





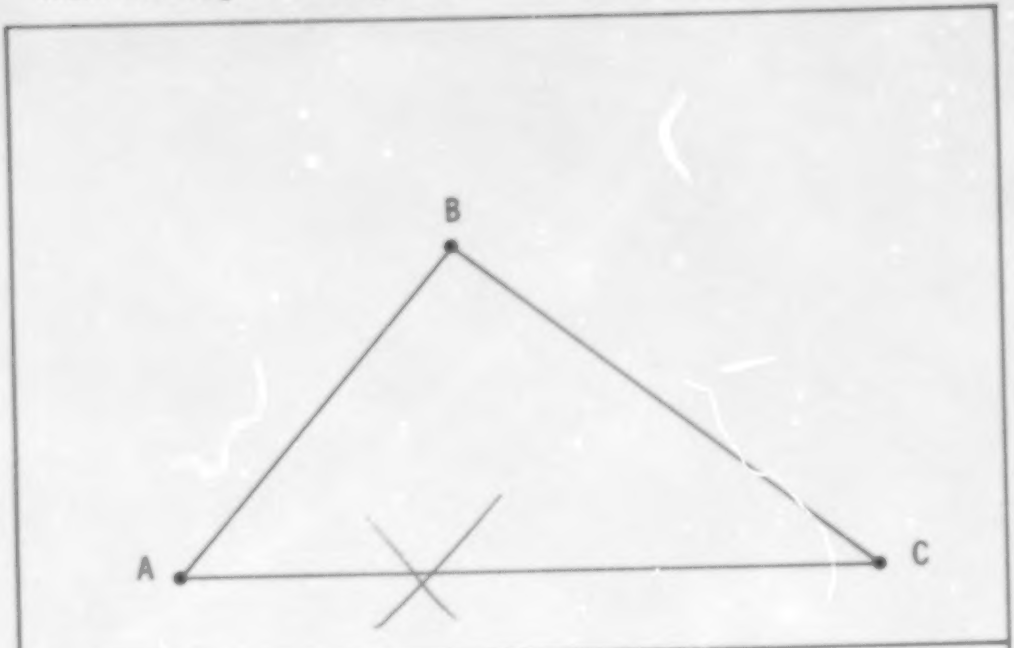
## Comparing Line Segments

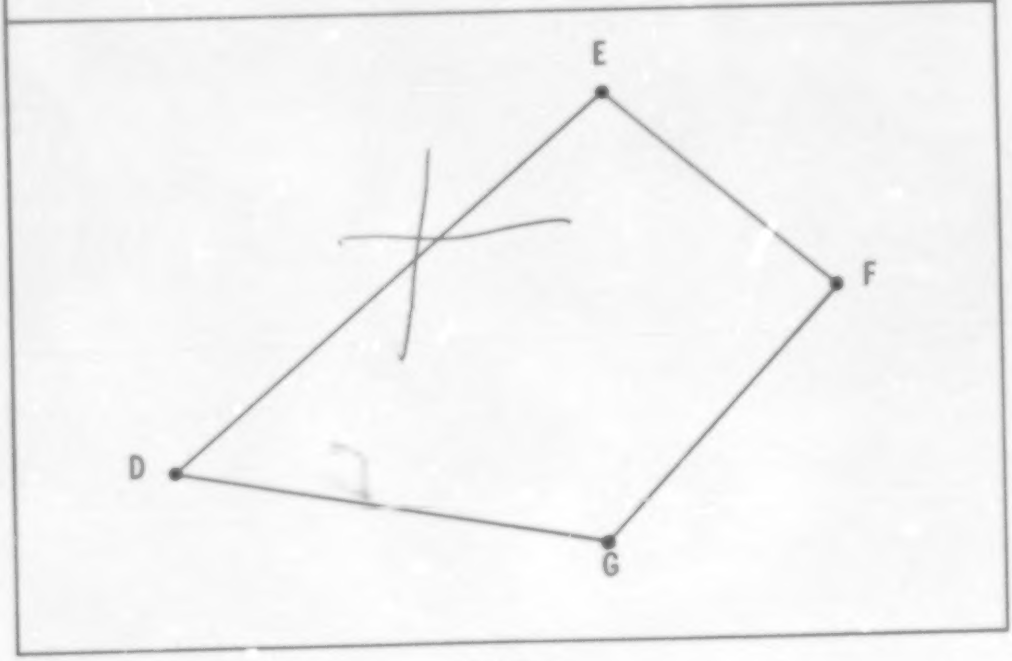
Mark the line segment that is shorter than the other one.



## Comparing Line Segments

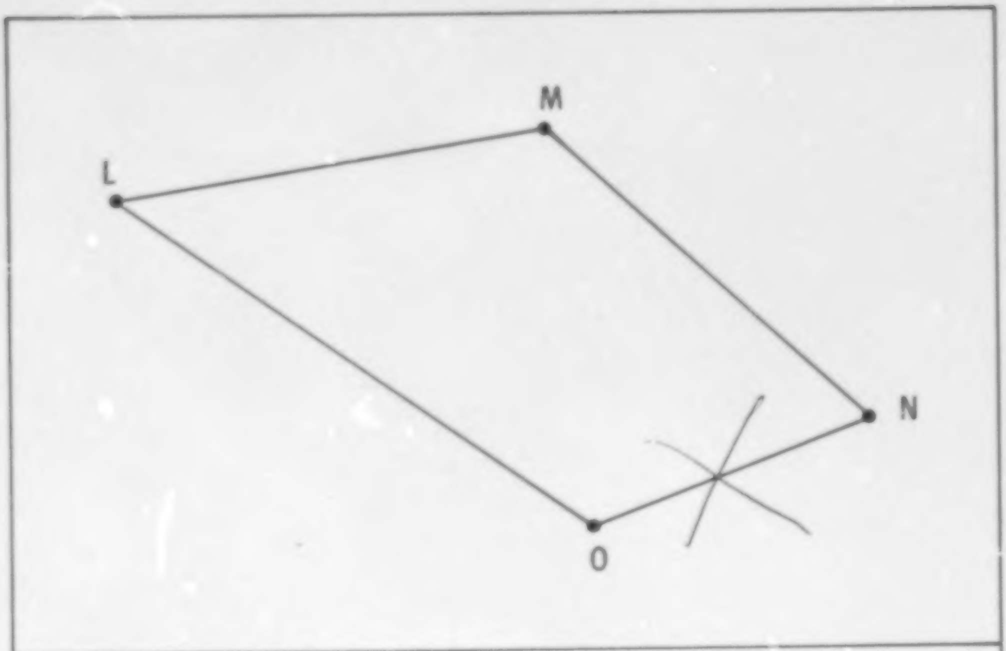
Mark the longest line segment.

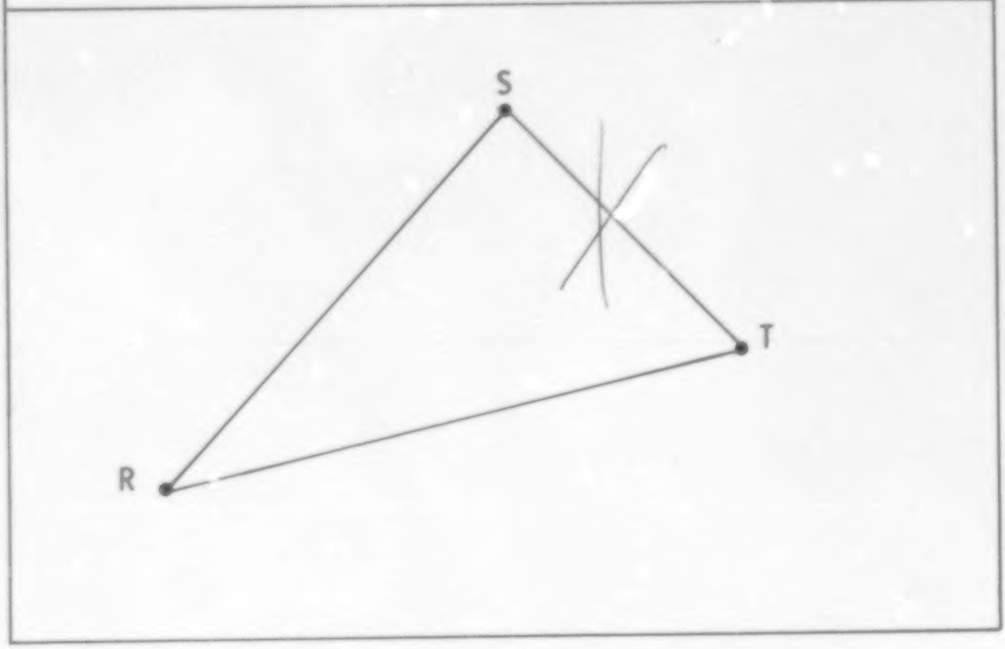




## Comparing Line Segments

Mark the line segment that is shortest.





## 10-3. MEASUREMENT OF LINE SEGMENTS

**OBJECTIVE:** To introduce the idea of measurement of a segment as the number of unit segments that can be laid end to end along it.

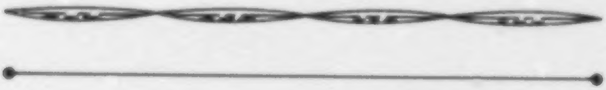
**VOCABULARY:** Unit segment, units, (review) length.

**MATERIALS:** Toothpicks, pieces of drinking straws, line segments drawn on paper.

### SUGGESTED PROCEDURE

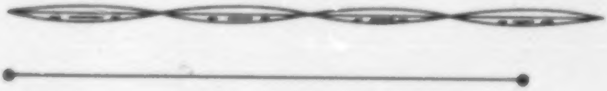
### PRE-BOOK ACTIVITIES:

Provide each child with a number of toothpicks of the same length. Make provision for a number of line segments to be measured. The endpoints should be clearly indicated. The exercise is to see how many of these toothpicks can be laid end to end along each line segment. Indicate that the toothpick is but one of many objects that we might use to measure line segments. We call the toothpick a *unit segment*. The length of the toothpick is one *unit*. The length of the line segment is 4 units. Have the children write the numerals 4 on their paper. Then continue with several other segments where the length is at least approximately the same as several toothpicks that are lined up.



The length is ___4___ units.

The next set of examples should be those where the unit segment does not fit exactly, as shown below. Have the children count the toothpicks and discover that the segment is between 3 and 4 toothpicks in length. Read the sentence and have the children write the numerals 3 and 4 where blanks are noted.



The length is between ___3___ units and ___4___ units.

Ask the children to check the examples again, this time using just one toothpick. Demonstrate how the toothpick is to be laid off and a mark made at the end each time so that the next measurement can be done carefully.

USING THE PUPIL'S BOOK, pages 274-275:

IDEAS: A line segment may be measured by repeatedly using a unit segment.

Have the children lay the toothpick repeatedly along the segments. Ask them to count the number of times the unit is used and to write the correct numerals where shown. Some of the examples may result in the last mark falling on the end of the line segment. In these cases explain that the number is not "between", but is the count of the unit segments.

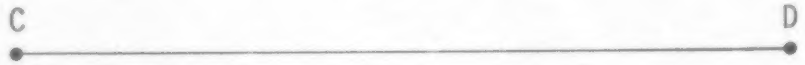
In all exercises, try to make sure that the pupils keep clearly in mind that the unit is a line segment. It is easy to have this idea obscured.

The exercises themselves will make clear the possible variety of units. Class discussion should crystallize the idea that for different units, a measurement has different numbers. To tell a length you need to tell not only the number of units but also to tell what unit is used. It should also be possible to develop the understanding that the smaller the unit, the greater the number needed for any particular measurement.

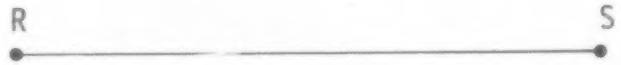
Measuring Line Segments

Use a unit segment to find each length.

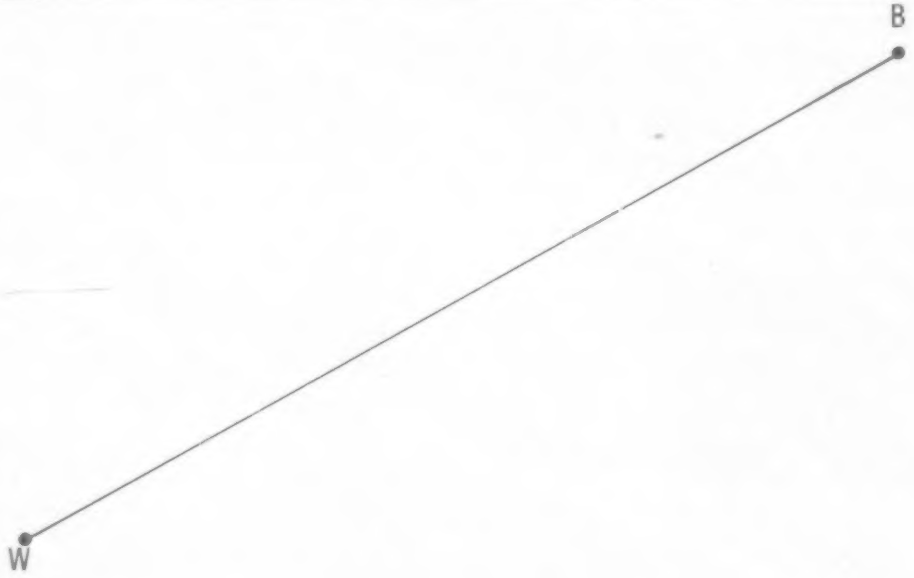
Answers depend upon toothpick used



The length of $\overline{CD}$ is between ______ and ______ units.



The length of $\overline{SR}$ is between ______ and ______ units.

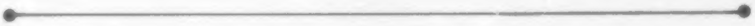


The length of $\overline{WB}$ is between ______ and ______ units.

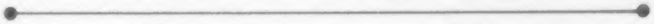
## Measuring Line Segments

Answers depend upon toothpick used

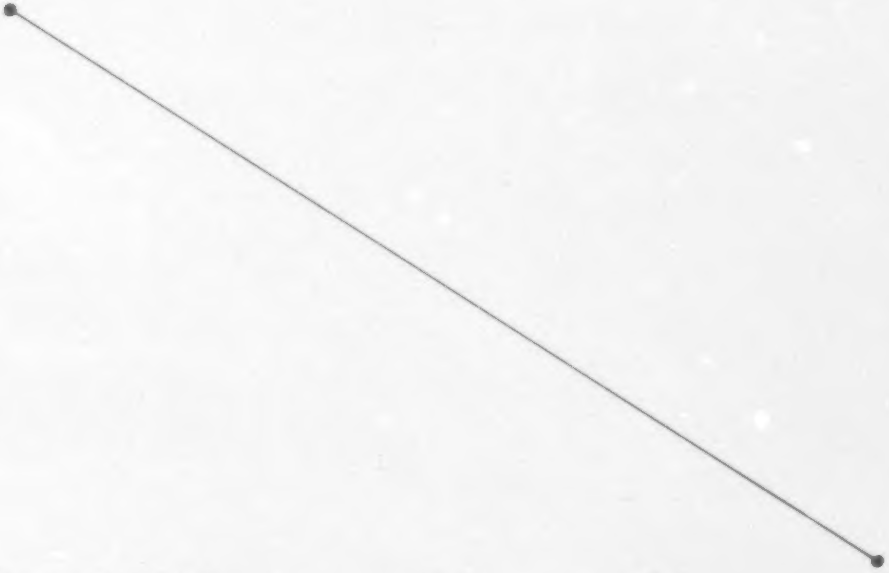
Use a unit segment to find each length.



The length is between ______ and ______ units.



The length is between ______ and ______ units.



The length is between ______ and ______ units.

FURTHER ACTIVITIES:

1. Provide each child with several different units (say pieces of drinking straws) and have him measure the same line segments with each unit. If straws are used, for example, there should be some designation attached to the different ones such as "long straw", "medium straw", and "short straw" so the pupil can describe his results as so many "short straws", etc. An alternative would be to use different objects as unit segments, such as pencil, chalk, etc.
2. Have different pupils measure the same line segment with different units. For example, draw a chalkline on the floor. Ask two children (with different sized feet) to see how many of their foot lengths it takes to walk from one end of the chalkline to the other. Instead of a chalkline you may wish to tape a piece of heavy cord to the floor.
3. Have the pupils invent their own units and use them. For example, how many of some child's hand-spans is it across the edge of the bookshelf?

## 10-4. CONSTRUCTION OF A RULER

OBJECTIVE: To introduce the idea of a scale as a measuring device.

VOCABULARY: (No new words.)

MATERIALS: Light cardboard straightedge (unmarked) perhaps a foot long, one for each pupil, some convenient unit segment (toothpicks or pieces of drinking straws), one for each pupil. To be convenient for handling, the units chosen should be around two inches or a little less.

### SUGGESTED PROCEDURE

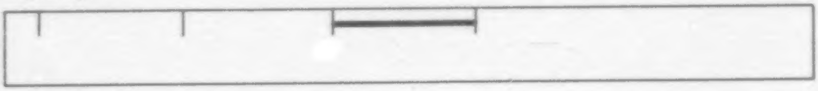
Give a straightedge to each pupil and ask the pupils to make a mark not far from the end. (This point is to be the zero point of the ruler. Note that the zero point is not at the end of the straightedge. In addition to being easier to identify, it avoids the problem that corners are always getting bent and dog-eared.)

Now ask each child to put his unit segment (a toothpick or whatever unit was used in Section 10-3) on the straightedge with one end on the initial mark and to mark the other end.

The piece of the straightedge is now a line segment one unit long.

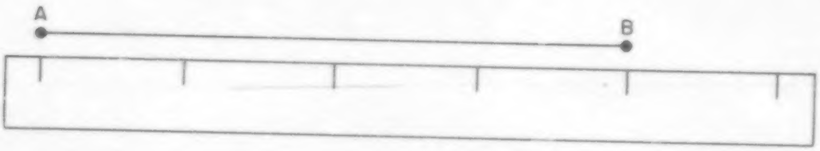
Now the unit segment can be laid down again,

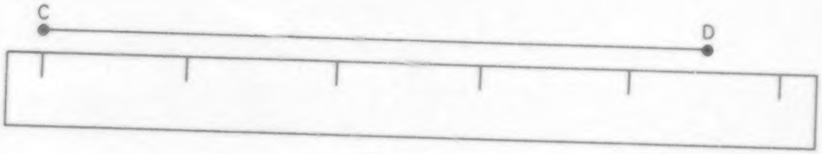
and again,



as often as the length of the straightedge allows. It is now easy to see that the marked straightedge shows line segments 1 unit long, or 2 units long, etc.

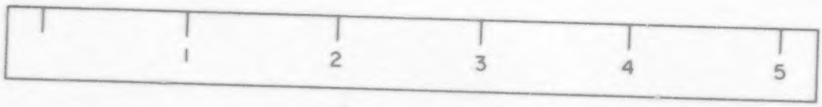
The straight edge in its present form can now be used for measuring line segments as shown below, where it is seen that the length of line segment $\overline{AB}$ is 4 units and the length of line segment $\overline{CD}$ is between 4 and 5 units.



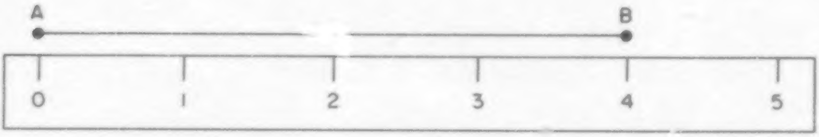


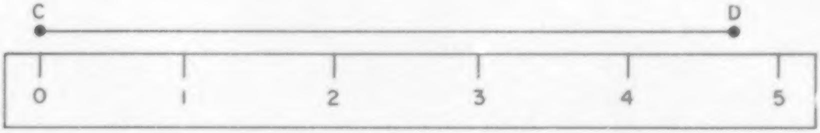
These numbers are found by counting the number of unit segments. The placing of the straightedge will need to be emphasized, i.e., the placing of the original mark at one end point of the line segment.

The next stage is to ask the children to label the marks on the straightedge. The idea is to put a 1 below the mark that was made the first time the unit was used, a 2 below the mark that was made the second time the unit was used, and so on. The instrument then looks like this.



Discussion should produce the suggestion that the original mark be labeled 0. The instrument is now complete and may properly be called a ruler. Indicate that the ruler shows part of a number line. In using it to measure line segments $\overline{AB}$ and $\overline{CD}$ as before, the numbering of the points produces a simplication.





The fact that in measuring line segment $\overline{AB}$ the point B is opposite the 4 mark shows that there were 4 copies of the unit segment between A and B. Thus, instead of looking back and counting the segments as was done before, the length of 4 units can be read directly from the ruler. Similarly, on line segment $\overline{CD}$, the point D is between the 4 and 5 marks; this shows that the length of line segment $\overline{CD}$ is between 4 and 5 units.

Practice may be given in measuring with this device, but it need not be stressed as the mechanics of using the ruler will be developed when standard units are introduced in a later book.